

Platforms, Tokens, and Interoperability

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Abstract

Digital money requires a ledger. By integrating this ledger with its other ledgers, a platform can enforce repayment of uncollateralized credit, beyond the ability of the banking sector. However, by controlling interoperability to other platforms' ledgers, an incumbent platform can “lock-in” customers and increase its market power. Open banking, which gives users control of interoperability, limits uncollateralized credit. Introducing CBDC as digital legal tender (on an isolated ledger) hurts credit extension, but enhances it when combined with an open architecture public ledger as a “smart CBDC”.

Keywords: Tokens, ledgers, interoperability, smart contracts, platforms, open banking, open architecture, financial inclusion, “smart CBDC”, “PlatFi”.

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1 Introduction

What makes “digital money” special? Unlike physical cash, it requires a digital ledger to record money holdings and transactions. Linking digital money ledgers with other databases creates the opportunity to have automatic execution and enforcement of complicated contracts. This opens up the possibility of a reorganization of the financial sector, which brings both opportunities as well as challenges. On the positive side, “BigTech” consumer platforms have started to provide tokens and/or expand credit to new groups (e.g. Alibaba, Meta, Amazon). Similarly, large supply chains and business-to-business platforms have started to move activities onto transparent ledgers, which enable them to offer tokens, automate operations, and extend trade credit (e.g. Corning, Emerson, Hayward, and other attempts at “Industry 4.0”). On the negative side, a platform can gain market power by controlling the interoperability to other platforms’ ledgers and “locking-in” customers. This raises important questions about how regulators should balance the potential synergies against the anti-competitive impact of non-interoperability. How integrated should ledgers be across different services as well as across competing platforms? Should the customers or platforms decide which information is shared across competing ledgers? Which design of a central bank digital currency (CBDC) competes optimally with private platforms’ digital token currencies?

In this paper, we model a supply chain with a profit maximising platform that provides a matching technology and a digital ledger, which it uses to create token money, collect payments, and write (“smart”) credit contracts. Our environment is populated by agents who arrive as *producers* without collateral, borrow funds from the platform using contracts written on the ledger, acquire input goods to produce, search as *sellers* for the opportunity to sell their inventory, then finally search as *buyers* for the opportunity to purchase consumption goods. The *incumbent private platform* faces competition from both a less technologically sophisticated *public marketplace* and from other potential *entrant private platforms* threatening to take over the market. We model competition with the former by allowing *buyers* and *sellers* to choose whether to search for trades on the incumbent private platform, making payments on the platform’s ledger using the platform’s currency (*tokens*), or the competitive public marketplace, where payments are made using the central bank currency (*money*). The platform is also able to use the transaction histories stored on the digital ledger to improve their matching service. Each seller, after receiving their different currencies, stores their wealth in a diversified currency fund at a conversion rate that reflects their currency’s relative usefulness as media of exchange. We model the contest between the incumbent platform and potential entrant platforms as a Stackleberg game, in which the incumbent platform moves first, the potential entrant platform responds, and then the buyers and sellers choose whether to switch to the entrant. The incumbent platform generates revenue through loan fees and “seigniorage revenue” when agents convert dollars into platform tokens. It attempts to maximise revenue by choosing a fee on loan contracts, the exchange rate at which tokens can be exchanged for dollars (the “interoperability” of currency), and the portability of ledger information to other ledgers (the “interoperability” of information).

We find two main results in a world without regulation. First, the incumbent platform is able

enforce uncollateralized credit because, by integrating its ledgers, it has information about trades, control of the payment system, and the ability to exclude agents from trading on their platform if they do not use their ledger. These synergies go beyond those of the banking sector and allow the platform to expand credit access. Second, the incumbent platform uses its control of the digital ledger to gain market power by restricting interoperability. A strength of our model is that it allows us to identify the different dimensions of interoperability and characterize how the trade-offs the platform faces when exploiting their intersecting impacts.

What is the incumbent platform’s trade-off between charging a higher lending fee versus offering a less favorable exchange rate? Both involve a similar general equilibrium trade off: they generate revenue but discourage sellers from coming to the private platform. This is because sellers dislike to trade where the automatically executed loan repayment is higher or where they have to accept payment in a currency with a worse conversion rate into the currency fund. However, the two policies have opposite effects on deterring entry. Decreasing lending fees makes it easier for the entrant to take control of the market because it can more cheaply undercut the loan fees charged by the incumbent. By contrast, increasing the exchange rate fee makes it harder for the entrant take control because they have to compensate agents for the cost of moving tokens off the incumbent platform. We refer to this as the “lock-in” effect of high exchange rate fees. We find that when agents are not price sensitive about their choice of trading platform, the incumbent platform prefers to attract agents by offering an enticing (possibly loss-making) lending fee and deter entry by setting very high exchange rate fees. However, as the agents become more price sensitive, the seller’s unwillingness to accept the less useful currency starts to dominate and the incumbent platform starts to reduce exchange rate fees and instead deter entry with low loan fees.

What are the incumbent platform’s trade-offs when deciding how easily information can be ported to the ledger of a potential entrant? To answer this question, note first that the different agents place different values on information on the ledger. In our model, buyers see their transaction histories as an asset (in addition to their token holdings) because they can be used to improve matching, while sellers see their contract information as a liability, since it enables repayment enforcement. The platform wants to restrict portability of information that give positive value to the agents, the transaction histories, so the buyers are “locked-in” to the platform. However, the platform wants to allow portability of information that gives negative value to the agents, like its debt obligations to the incumbent platform. If the incumbent platform were not to share this information, debtor agents have an incentive to default on her debt and switch over to the entrant platform (with a fresh start). Without information sharing this would act like a “kick-out” effect. We can interpret these portability policies as similar to those we observe in the current banking system: banks keep transaction histories secret, while they share credit histories via credit registries.

Another set of results concerns regulatory guidance. First, we show that “open-banking” regulation shuts down the uncollateralized credit market. “Open banking” grants agents the right to decide whether to pass on their information to other platform ledgers. Under this regulation, agents makes the opposite information portability choices to the platform. Buyers choose to port

their transaction histories since they help the new platform improve its matching technology while sellers do not port their contract information since it allows them to more easily default if they move to the entrant platform. Of course, the incumbent platform anticipates this and consequently extends less uncollateralized credit in the first place.

Second, we show that total interoperability also shuts the uncollateralized credit market. In principle, the regulator could impose both perfect information portability and one-for-one exchangeability of tokens. Under such a regulatory framework any “walled gardens”, in which agents are trapped, are broken down.¹ Unlike in the open banking case, this means that contracts can be enforced on the entrant platform. However, the incumbent platform is none-the-less unwilling to make loans because it loses value when it bargains with the entrant platform over how to resolve the contracts when sellers move platforms. Ultimately, in the equilibrium in the contested markets, the value of the incumbent platform is the cost the new platform bears for taking control of the market and, under full interoperability, the new entrant can take over the market at negative cost because they can get part of the loan repayment when sellers move. This means that the incumbent platform gets a negative value from setting up the credit market and will not do it.

Finally, we show that whether CBDC eliminates or enhances synergies for uncollateralized credit provision depends on how CBDC is implemented. We study the introduction of a legal tender CBDC that agents have the right to use on any platform. In particular, agents can search on the private platform but receive their payments in CBDC. This allows them to pretend that they have been unable to sell their output goods and default on the credit contract (while cashing in secretly using CBDC). Consequently, the credit market shuts down. Luckily, this does not have to be the case if a “smart CBDC” ledger design is employed. A smart CBDC arrangement offers a fully-interoperable public ledger (“open-architecture”) which integrates with various platform ledgers—including the public marketplace’s ledger. Smart CBDC enables credit extension, since the smart contract information situated at the incumbent platform can also be used to enforce contracts on the public marketplace and the potential entrant platform ledger. Conceptually, there is positive externality from the provision of “smart” CBDC ledger. The government has an advantage in getting their digital currency used across all platforms and offers a programmable “open architecture ledger”. This ensures that contracts can be enforced across a wide range of digital markets. However, such a “meta-ledger” requires special governance to resolve privacy issues—which goes beyond the scope of this paper.

Literature Review

This paper builds on literatures on platform competition, market design, industrial organization, international trade, monetary economics, and the burgeoning literature on CBDC.

First, we relate to the growing field of digital currencies. Many of these papers have focused on decentralised digital currencies as e.g. cryptocurrencies (e.g. [Fernández-Villaverde and Sanches \(2018\)](#), [Benigno et al. \(2019\)](#), [Abadi and Brunnermeier \(2018\)](#), [Schilling and Uhlig \(2019\)](#)). Others

¹See [Bank for International Settlements \(2021\)](#).

analyze central bank digital currency (e.g. [Fernández-Villaverde et al. \(2020\)](#), [Keister and Sanches \(2019\)](#), [Kahn et al. \(2019\)](#)). This paper studies the impact of CBDC, especially if enforced as legal tender, on the interoperability of platforms and tokens.

There is also a long literature on currency competition (e.g. [Hayek \(1976\)](#), [Kareken and Wallace \(1981\)](#), [Brunnermeier and Sannikov \(2019\)](#)). Instead of focusing on international currency competition, in this paper tokens of platforms and public money compete with each other.

Finally, a connection can also be drawn to the work initial coin offerings (e.g. [Chiu and Wong \(2020\)](#), [Cong et al. \(2020\)](#), [You and Rogoff \(2019\)](#), [Catalini and Gans \(2018\)](#), [Garratt and Van Oordt \(2019\)](#)), on currency management (e.g. [Gans and Halaburda \(2015\)](#), [Chiu and Wong \(2020\)](#)) and, more generally, to the money search literature studying how government money (or competition amongst intermediaries) provides an outside option that decreases intermediary bargaining strength (e.g. [Lagos and Zhang \(2019\)](#), [Lagos and Zhang \(2020\)](#), and [Chiu et al. \(2019\)](#)).

Second, our model relates to the literature on platforms and intermediaries, as e.g. [Rubinstein and Wolinsky \(1987\)](#), [Rochet and Tirole \(2003, 2006\)](#), [Spulber \(1999, 2019\)](#) as well as [Fudenberg and Tirole \(2000\)](#), in which an incumbent platform also competes with potential entrant platforms. Importantly, we extend the market design with token currency design features. For the discrete choice of agents across platforms we borrow modeling tools from the industrial organization and international trade literature, e.g. [McFadden and Train \(2000\)](#), [Eaton and Kortum \(2002\)](#), [Redding and Weinstein \(2015\)](#). A key difference in our paper, is that our network externalities arise endogenously through the interaction of the different platform services rather than exogeneously through a matching function.

We structure the paper in the following way. Section 2 defines and characterizes equilibrium for a given set of platform policies. Section 3 solves the platform’s digital ledger design problem. Section 4 studies the impact of the platform’s different interoperability choices. Section 5 discusses CBDC and other implementations of interoperability regulation. Section 6 concludes.

2 Environment

Time is continuous with infinite horizon. There is one perishable input good and one consumption good. At any time, the economy is populated by a continuum of traders and a private platform.

Production supply chain and preferences: Each agent follows a “life-cycle” as they move through a “supply-chain”.² Agents arrive at rate λ^a without resources but with a production opportunity to immediately convert $1/z$ units of other agents’ labor into 1 consumption goods. After producing, they start searching for an opportunity to sell their inventory of consumption goods. We refer to these traders as “sellers”. After selling their production, the agents start providing labour to producers at unit marginal disutility and start searching for an opportunity to purchase consumption

²We model a “life-cycle” for convenience. Qualitatively, the results would hold so long as there are some agents in the economy who are borrowing constrained.

goods. We refer to these agents as “buyers”. Once an agent finds a buying opportunity, they get log utility, $u(c) = \log(c)$, from consuming $c \geq 0$ consumption goods and exit. They do not get utility from consuming at other times.³ All traders have a discount ρ throughout their life.

Labor market, goods market, and matching frictions: The labor market is competitive and frictionless. By contrast, agents must search to trade consumption goods. At any point in time, there are two platform technologies for matching buyers and sellers, indexed by $\mathcal{P} \in \{0, 1\}$. Platform $\mathcal{P} = 0$ is not controlled by any agent and is referred to as the “public” marketplace. Platform $\mathcal{P} = 1$ is controlled by a profit maximizing organization and is referred to as the “incumbent private” platform.⁴ In addition to pecuniary benefits from trading on a platform, each time period, t , for each platform, $\mathcal{P} \in \{0, 1\}$, each agent, i , gets an idiosyncratic, independent amenity draw for trading on that platform.⁵ For sellers, the draw is distributed according to $\zeta_t^{s\mathcal{P}i} \sim \text{Gu}(1/\xi^s, -(1/\xi^s)\mathcal{E})$, where \mathcal{E} is the Euler–Mascheroni constant. For buyers, the draw is distributed according to $\zeta_t^{b\mathcal{P}i} \sim \log(\zeta_t^{\mathcal{P}}) + \text{Gu}(1/\xi^b, -(1/\xi^b)\mathcal{E})$, where $\zeta_t^{\mathcal{P}}$ is a platform specific component that characterizes the average service quality provided by the platform.⁶ We do not impose a physical interpretation on the amenity values but they could be modeled as idiosyncratic search costs or good quality.⁷ We allow $\zeta_t^{\mathcal{P}}$ to vary with the state of the economy to allow for potential economies of scale in the matching. Each period, after observing their amenities, buyers and sellers choose on which platform to search and find trading opportunities at rates λ^s and λ^b respectively. A competitive market then opens on each platform amongst the buyers and sellers who found trading opportunities.⁸

Currencies and Markets: At any point in time, there are two currencies circulating in the economy, indexed by $\mathcal{C} \in \{0, 1\}$. Currency $\mathcal{C} = 0$ denotes “money” issued and maintained by the government and currency $\mathcal{C} = 1$ denotes digital “tokens” issued by the private platform. The platform lets agents convert dollars into tokens at an exchange rate of 1 but imposes an exchange rate of $\epsilon \leq 1$ for converting tokens into money. The digital tokens are backed by holdings of government

³We impose this assumption to prevent agents from consuming the goods that they produce. Alternatively, we could assume that agents produce different types of consumption goods or as the buyers only needing to consume after they have produced.

⁴We use the term incumbent because we will later consider the possibility that a new platform enters the market. The entrant platform will be indexed by $\mathcal{P} = 2$.

⁵We introduce idiosyncratic risk in order to avoid “bang-bang” solutions to the platform choice problem. This is analogous to assuming a CES preference function across the platforms.

⁶In principle, we could introduce the trading advantage on either the buyer or seller side of the market. We introduce it on the buyer side because we later model the platform as acquiring information from when agents sell to improve their service quality when they buy.

⁷For the cost interpretation, note that the Gumbel distribution takes values across the real line and so $\zeta^{b\mathcal{P}i}$ would represent a normalized cost. For the good quality interpretation, observe that we can write the total utility of a buyer receives as: $\log(e^{\zeta^{\mathcal{P}i}} \zeta^{b\mathcal{P}} c)$ and so $e^{\zeta^{\mathcal{P}i}} \zeta^{b\mathcal{P}}$ is essentially scaling the utility that the buyer gets from the good they consume.

⁸The search literature often considers one-to-one matching and bargaining. We instead consider segmented competitive markets because we think it is a closer approximation to how trade is organized on large platforms such as Amazon or Alibaba.

currency so they are a form of “stablecoin”.⁹ There is no other market for trading currencies.¹⁰ We start with the assumption that sellers accept dollars in the public marketplace but the platform mandates that tokens must be accepted when trading on the private platform.¹¹

Ledgers, information, and contracts: The controller of the private platform organizes their digital tokens through a centralized, transparent, and programmable ledger. In addition to containing token accounts, the ledger also contains information about past transactions and instructions for automatically executing the terms of contracts, which we refer to as “smart” contracts.¹² The incumbent platform does not intermediate transactions but instead organizes all financial arrangements through the ledger. This means that it writes contracts through the ledger, provides information about trades on their platform to the ledger, and forces traders on their platform to use their ledger to make their payments in tokens.

The incumbent platform writes and commits to contingent “trade-credit-like” contracts with agents. Under a contract, the platform supplies resources to the producer so they can purchase input goods. In return, sales revenue is divided between the producer and the platform with the producer keeping a fraction κ and the platform keeping the remaining fraction $1 - \kappa$. We interpret κ as combining both a “loan fee” and a “markup” for trading on the platform.¹³ Producers lack commitment and do not have collateral. There is no legal system that can enforce contracts, which we interpret as an international credit market or an economy with weak government institutions. However, the contract can be automatically enforced so long as information about the transaction is provided to the digital ledger (there is information to activate the smart contract) and payments are carried out through the digital ledger (the smart contract can access the resources being used for payment).¹⁴ We consider a richer contract design problem in section 5.3.

The platform can use the information on the ledger to improve the matching technology. We

⁹We do not focus on the platform balance sheet financial stability is not the focus of the paper. However, formally, this arrangement is implemented in the following way. If an agent gives a dollar to the platform, then the platform holds ϵ dollars in a reserve and pays out $1 - \epsilon$ dollars as dividends. If an agent brings back a token, then the platform pays ϵ from their reserves and destroys the token.

¹⁰We assume that the private platform prevents their tokens being traded on digital token centralized exchanges and we assume that side trading via decentralized online forums is not possible. This is a strong assumption but our results would still hold qualitatively if side trading was allowed but frictional. We return to this assumption when we discuss CBDC in subsection 5.3.

¹¹We relax this assumption in section 5.3 when we endogenize seller acceptance choice and introduce legal tender restrictions.

¹²We use the term “smart” contract because the setup is motivated by the DeFi industry. However, there is nothing decentralized about the maintenance of the ledger. The platform operator controls the updates to the ledger. A “smart” contract is simply a contract that is automatically executed by the ledger when an event specified in the contract occurs.

¹³In principle, we could model the platform as charging a separate loan fee and markup fee that the seller must post in advance to search on the platform. However, since the seller will ultimately only repay the loan when they trade on the platform, there is no difference between a markup and a loan fee in this environment.

¹⁴We assume that the platform is able to link the identities of sellers to the identity of producers. In this simple model, no seller has inventory without a debt contract to the platform so the platform can simply do this by forcing sellers to post not only an offer to sell a good but also the debt contract that they will repay. In a more complex model where some agents have inventory without a debt contract, the platform would need to incentivize agents to reveal their identity.

model this by assuming that the platform specific component of the amenity value satisfies $1 + \zeta(\varphi_t) \geq 1$, where φ_t is the fraction of current buyers who sold on the private platform and we impose that ζ is weakly increasing in φ .

Currency fund: There is a fund into which buyers can pool their currency holdings. The exchange rate for purchasing one share in the fund with currency \mathcal{C} is given by $q^{\mathcal{C}}$, where we assume $q^{\mathcal{C}}$ is the marginal value of an additional unit of currency \mathcal{C} to the existing shareholders in the fund. When a buyer get a buying opportunity, they take a portfolio of currencies from the fund proportional to the funds' relative holdings of money and tokens. Once they have taken money out of the fund, they cannot trade again with the fund. In this sense, they face a “cash-in-advance” constraint when trading. Conceptually, we can think of the fund as selling agents an “average” token for $q^{\mathcal{C}}$ units of currency \mathcal{C} so that agents can share some currency risk. The fund is also the residual owner of the platform and values dividends in currency \mathcal{C} using the fund entry exchange rate $q^{\mathcal{C}}$. All wages and dividends brought to the fund are allocated to buyer proportional to claim on the fund.¹⁵ Wages are paid in a mixture of tokens and dollars, proportional to the funds holdings of tokens and dollars.¹⁶

Private Platform Entry: At any time t , a new platform, indexed by platform 2, may enter and contest the market. In order to successfully dethrone the incumbent, the new platform must convince agents to move the new platform. Buyers and sellers do not get an idiosyncratic shock when comparing between the two private platforms. However, the entrant platform service amenity depends on how portable the incumbent platform chooses to make the information on the ledger, ι . Portability $\iota = 0$ indicates that no information can be moved off the platform while $\iota = 1$ indicates that the entrant platform can access all information on the platform. This means that the average matching technology for the entrant when they contest the market is given by $\zeta(\iota\varphi)$ and the fraction of contracts that can be enforced on the new ledger is ι . If the entrant does successfully capture the market, then the entrant and the incumbent bargain over the orphaned, enforceable contracts with the entrant having bargaining power α . We discuss the details of the entry game when we set up the entrant and incumbent problems in section 4. We restrict attention to equilibria where the incumbent successfully deters entry so the entry game acts as “off-equilibrium path” threat.

Timing: At time 0, the incumbent platform commits to a collection of policies for loan fees, exchange rate fees, and information portability, $\{\kappa, \epsilon, \iota\}$. Given these policies, the timing within the period is then given by:

1. *Production and search:* Producers arrive, contract with the platform, purchase input goods and produce inventory. Buyers take their portfolio from the fund. Buyers and sellers observe

¹⁵We make this assumption to ensure aggregation. We do not attempt to discuss issues of inequality amongst the buyers.

¹⁶We make this technical assumption so it is only the agents decision about which platform to trade on that change the fund's portfolio.

their amenity from trading on each platform, then choose on which platform to search under a belief about whether a new platform will enter.

2. *Potential entry:* Entrant decides whether to enter the market with policy κ^2 . If entry occurs, then buyers and sellers who chose to trade on the incumbent private platform can switch to the entrant private platform.
3. *Trading opportunities are realized and contracts are resolved.* All agents with trading opportunities participate in a competitive market. The platform that has no trade leaves the market. Platforms bargain over orphaned contracts and then resolve contracts.

The environment imposes a parsimonious collection of frictions to model different roles for the platform, different types of ledger information, and dynamic “lock-in” effects. We discuss how the model assumptions generate these forces below.

Discussion of what makes “digital” money special: We argue that the key difference between digital money and physical cash is that digital money requires a digital ledger whereas cash does not require an explicit ledger. In our model, the platform combines a digital token ledger with their other ledgers that record seller posts, buyer bids, and agent trades. In this sense, the platform is able to create an integrated ledger with the relevant trade information and token accounts. The platform is then able to code contracts onto the ledger that use trade information to automatically trigger loan repayments. So, the platform can enforce contracts so long as they can incentivise sellers to trade through their ledger. This potential synergy between integrating token money ledgers with supply chain trade ledgers is the key force that we seek to explore in this paper. In section 6, we discuss other potential market arrangements that could generate similar synergies, such as if an independent digital ledger contracts with a platform to get access to trade information.

Discussion of platform role: The platform provides three services to this economy: uncollateralized credit extensions to producers, a matching technology in the consumption goods market, and a token currency that can be used for storage:

- (i) The platform is able to enforce uncollateralized credit because they can “exclude” agents from using their matching service if the agents don’t use the platform’s token ledger, where the repayment of debt contracts automatically occurs. In this sense, the bundling of a matching service with a digital ledger gives the platform the exclusion power required to enforce contracts. We model exclusion in a parsimonious way as “one-period” exclusion from trading on the private platform. However, qualitatively the results would not change if we built a more quantitatively realistic model where agents live longer and face longer exclusion periods.
- (ii) The platform provides a differentiated matching service. As in other discrete choice models, the scale parameters, ξ^b and ξ^s , determine the elasticity of substitution between the matching

service offered by the private platform and the public market place. The ratio $\zeta^1/\zeta^0 = \zeta(\varphi)$ characterizes the relative average advantage (or disadvantage) of the private platform. The network effect from information is that $\zeta(\varphi)$ increase as a greater fraction of agents trade on the private platform ($\varphi \uparrow$) because the platform has more information with which to customize services.

- (iii) Both currencies are valuable in the model because the buyers need a store of value and because there are cash-in-advance constraints for consuming on the platforms. However, the lower the incumbent sets the exchange rate, ϵ , the worse tokens are as a medium of exchange and so the harder it is for the incumbent to persuade sellers to accept the tokens.

Discussion of tokens and other information: At an abstract level, a ledger stores information. We divide that information into (i) tokens that summarize the net-wealth position of the agent and (ii) other information such as transaction histories and contract instructions. The distinction is that tokens can be exchanged amongst agents while the other information can be exchanged across ledgers. In our baseline model, the government money provides competition with tokens while the entrant ledger provides competition for broader information on the ledger. In section 5.3, we consider central bank digital currency (CBDC) arrangements in which the the government provides a ledger that competes as both a token provider and a store of information.

Discussion of “lock-in” effects and other dynamics: We model a infinite horizon, two-sided market to capture the platform’s ability to use currency issuance to “lock-in” customers and generate meaningful money demand functions. If more sellers accept tokens, then the economy (and so the fund) has more tokens and so buyers take more tokens to the goods markets. The platform can then impose token “exit costs” for converting tokens into dollars so that agents are more likely to trade on the incumbent platform where tokens are accepted. In this sense, the incumbent can decrease ϵ to deter entry but balance this with keeping the currency valuable as a medium of exchange so that sellers are willing to accept it.

The other assumptions are primarily technical restrictions that help us to solve for equilibrium in closed form and so make the platform competition problem tractable. We believe this is the most parsimonious model that can generate the economic forces described above.

3 Market Equilibrium

In this section, we characterize equilibrium for a given collection of platform policies without the possibility of new platform entry. This will allow us to illustrate the connections between the three roles that the platform is playing in the economy. In the subsequent sections, we study how a platform chooses policies disciplined by the threat of new platform entry.

3.1 Agent Problems

Currency holder (“buyer”) problem: Let $A_t := M_t^0 + M_t^1$ denote the total currency in the buyers’ fund, where M_t^C denotes the units of currency C in the fund. Let $\eta_t := M_t^1/A_t$ denote the fraction of private tokens in the fund.¹⁷ Let a_t^i denote agent i ’s claim on the fund, which evolves according:

$$da_t^i = \hat{\mu}_t^a a_t^i dt = (w_t + \pi_t) a_t^i dt$$

where w_t is wage income per currency unit and π_t is the net income from platform dividends and seller entry exchange rates per currency unit. Taking prices and laws of motion for η_t and A_t as given, buyer i chooses on which platform to search to solve problem (3.1) below:

$$\begin{aligned} V_t^{bi}(a_t^i) &= \max_{\mathcal{P}, c^{\mathcal{P}}} \left\{ \mathbb{E}_t \left[e^{-\rho\tau} (\zeta_\tau^{b\mathcal{P}i} + u(c_\tau^{\mathcal{P}})) \right] \right\} \quad s.t. \\ p_\tau^{\mathcal{P}} c_\tau^{\mathcal{P}} &\leq ((1 - \eta_\tau) + \epsilon^{1\mathcal{P}} \eta_\tau) a_\tau^i, \quad \forall \mathcal{P} \in \{0, 1\} \\ &\text{Laws of motion for } a_t^i, \eta_t, \text{ and } A_t \end{aligned} \quad (3.1)$$

where $\mathcal{P}_t \in \{0, 1\}$ is the platform on which the buyer is searching at time t , $\zeta_t^{b\mathcal{P}i}$ is the amenity benefit of search on platform \mathcal{P} at time t , the buying opportunity time, τ , arrives at rate λ^b

Inventory holder (“seller”) problem: Taking prices as given, seller j with inventory n_t^j choose on which platform to search to solve problem (3.2) below:

$$V_t^{sj} = \max_{\mathcal{P}} \left\{ \mathbb{E}_t \left[e^{-\rho\tau} (\zeta_\tau^{s\mathcal{P}j} + V_\tau^{bi}(\kappa^{\mathcal{P}} p_\tau^{\mathcal{P}} n_\tau^j)) \right] \right\} \quad (3.2)$$

where $\mathcal{P}_t^j \in \{0, 1\}$ is the platform on which the seller is searching at time t , the buying opportunity time, τ , arrives according to a Poisson process with rate λ_t^s , and $\kappa^{\mathcal{P}}$ is the fraction that the seller keeps after any default if they sell on platform \mathcal{P} . Since contracts are only enforceable in our baseline model when sellers trade on the private platform, we have $\kappa^0 = 1$ and $\kappa^1 = \kappa$.

Producer problem: At the time of entry, the agent chooses whether to contract with the platform to acquire input goods or not enter the market. Their outside option is zero so potential producers solve problem (3.3) below:

$$\max\{V_t^{si}, 0\} \quad (3.3)$$

3.2 Recursive Market Equilibrium

We set up the market equilibrium recursively. The aggregate state variables are $Z_t = \{\varphi_t, \eta_t, N_t, A_t\}$, where φ_t is the fraction of current buyers who sold on the private platform, η_t is the fraction of fund currency in tokens, N_t is total inventory in the economy, and A_t is total currency in the

¹⁷Where convenient, we will use $\eta_t^C := M_t^C/A_t$ to denote the fraction of currency C .

fund. To keep the problem analytically tractable, we restrict attention to equilibria with constant platform policies, constant fund portfolio shares, and constant prices. We refer to such equilibria as “stationary”. Formally, this is defined in the following way.

Definition 1 (Recursive Stationary Market Equilibrium). Given platform policies, $\{\kappa, \epsilon, \iota\}$, a recursive, stationary market equilibrium is a collection of consumption goods price functions, $\{p^{\mathcal{P}}(Z)\}_{\mathcal{P} \in \{0,1\}}$, fund exchange rates, $\{q^{\mathcal{C}}(Z)\}_{\mathcal{C} \in \{0,1\}}$, a buyer value function, $\{V^b(a, Z)\}$, buyer choice functions, $\{\mathcal{P}^b(a, Z), x^{b\mathcal{P}}(a, Z)\}_{\mathcal{P} \in \{0,1\}}$, a seller value function, $V^s(Z)$, a seller choice function, $\{\mathcal{P}^s(Z)\}_{\mathcal{P} \in \{0,1\}}$, and producer entry decisions such that:

- (i) Given consumption goods prices, the buyer value function and choices solve the HJBE associated with problem (3.1), the seller value function and choices solve HJBE associated with problem (3.2), and producers solve (3.3),
- (ii) The fund is indifferent about new agents entering the fund, and
- (iii) The consumption goods markets clear on each platform $\mathcal{P} \in \{0, 1\}$.

The problem is sufficiently tractable that we can characterize the value functions of the buyers and sellers in a stationary equilibrium. We do this in theorem 1 below.

Theorem 1. *Suppose that the economy is in a stationary equilibrium with state variables $Z = \{\varphi, \eta, N, A\}$. Given prices, the value function of the buyer is given by:*

$$V^b(a, Z) = \beta^b \left(\log(\bar{v}^b(\varphi, \eta, \epsilon)a) + \frac{\mu^a}{\rho + \lambda^b} \right), \quad \text{where}$$

$$\beta^b = \frac{\lambda^b}{\rho + \lambda^b}, \quad \bar{v}^b(\varphi, \eta, \epsilon) = \left(\sum_{\mathcal{P}} \frac{1}{2} \left(\frac{(1 + \zeta^{\mathcal{P}}(\varphi))(1 - \eta + \epsilon^{1\mathcal{P}}\eta)}{p^{\mathcal{P}}} \right)^{\xi^b} \right)^{1/\xi^b},$$

and $\epsilon^{\mathcal{C}\mathcal{C}'}$ denotes the conversion rate from currency \mathcal{C} to currency \mathcal{C}' . Given prices, the value function of the seller is given by:

$$V^s(n, Z) = \beta^s V^b(\bar{v}^s n, Z), \quad \text{where}$$

$$\beta^s = \frac{\lambda^s}{\rho + \lambda^s}, \quad \bar{v}^s = \left(\sum_{\mathcal{P}} \frac{1}{2} \left(q^{\mathcal{P}} \kappa^{\mathcal{P}} p^{\mathcal{P}} \right)^{\xi^s} \right)^{1/\xi^s},$$

The fraction of buyers and sellers that search on platform 1 is given by:

$$\phi^{b1} = \frac{1}{1 + \left[\left(\frac{1 - \eta + \epsilon\eta}{1 + \zeta(\varphi)} \right) \left(\frac{p^1}{p^0} \right) \right]^{\xi^b}} \tag{3.4}$$

$$\phi^{s1} = \frac{1}{1 + \left[\left(\frac{q^0}{\kappa q^1} \right) \left(\frac{p^0}{p^1} \right) \right]^{\xi^s}}. \tag{3.5}$$

Proof. See appendix A.2.1. □

We can interpret buyer and seller problems in terms of their “claim” on the ledger along both dimensions that we consider this paper. The value function for buyers depends upon $\bar{v}^b(\varphi, \eta, \epsilon)a$, where the first term is the expected marginal value of wealth and the second term is the wealth held by the buyer. The buyers have a “positive” token position on the ledger through their token holdings ηa . They also have a “positive” position with respect to transaction information on the ledger because an increase in transaction volume increases $\zeta(\varphi)$ and so the value of their wealth.

The value function for sellers also depends upon $\bar{v}^b(\varphi, \eta, \epsilon)$ because the sellers eventually become buyers. However, it also depends upon the expected sales revenue that the seller will receive: $\bar{v}^s n$. The expected marginal value of inventory, \bar{v}^s , can be written as:

$$\bar{v}^s = \left(\sum_{\mathcal{P}} \left(q^{\mathcal{P}} (p^{\mathcal{P}} n + a^{\mathcal{P}}) \right)^{\xi^s} \right)^{1/\xi^s}$$

where $a^{\mathcal{P}} = -(1 - \kappa^{\mathcal{P}})p^{\mathcal{P}}n$ is the net financial wealth of a seller who chooses to sell on platform \mathcal{P} . In this sense, we can see that sellers have a “negative” token position on the ledger. Ultimately, we will show that they also have a “negative” position with respect to the information on the ledger because when information is more portable their contracts can more easily be enforced on a new entrant platform and so κ is lower.

We define the value of the fund to be equally weighted value of all buyers in the fund. So, using the solution to the buyer decision problem, we can characterize the value of the “fund”. This is done in corollary 1 below.

Corollary 1. *Suppose that the fund has a mass of buyers, B , a distribution of buyer claims, $\{a^i\}$, and a fraction of wealth in tokens, η . Then, the value of the fund:*

$$V^f(\{a^i\}, \varphi, \eta, B) := \int_0^B V^b(a^i, \eta, B) di = \beta^b B \left(\log(\bar{v}^b(\varphi, \eta)) + \frac{1}{B} \int_0^B \log(a^i) di + \frac{\hat{\mu}^a}{\rho + \lambda^a} \right)$$

Using the solution to the agent decision problems and the value of the fund, we can characterize the equilibrium price ratios. This is done in Theorem 2 below.

Theorem 2. *Suppose that the economy is in a stationary equilibrium with state variables $\{\varphi, \eta, A, N\}$. Then, the equilibrium goods price ratio and fund exchange rate ratio solve:*

$$\frac{p^1}{p^0} = \left[\left(\frac{1 + \zeta(\varphi)}{1 - \eta + \epsilon\eta} \right)^{(1+\xi^b)} \left(\frac{q^0}{\kappa q^1} \right)^{\xi^s} \right]^{\frac{1}{1+\xi^b+\xi^s}} \quad (3.6)$$

$$\frac{q^1}{q^0} = \frac{\epsilon(1 - \eta + \epsilon\eta)^{\xi^b-1} + \left(\frac{(1+\zeta(\varphi))p^0}{p^1} \right)^{\xi^b}}{(1 - \eta + \epsilon\eta)^{\xi^b-1} + \left(\frac{(1+\zeta(\varphi))p^0}{p^1} \right)^{\xi^b}} \quad (3.7)$$

Proof. See appendix A.2.4. □

This equilibrium characterization illustrates how different platform policies attract buyers and

sellers and impact the price ratios. An increase in the private platform's matching advantage, $\zeta(\varphi)$, or a decrease in the exchange rate for moving tokens backs to dollars, ϵ , lead to an increase in the fraction of buyers choosing the private platform and so an increase in the equilibrium price ratio, p^1/p^0 . An increase in fraction of sales revenue that the sellers can keep when they trade on the private platform, κ , or an increase in relative fund exchange rate for bringing tokens, q^1/q^0 , lead to an increase in the fraction of sellers choosing the private platform and so a decrease in the equilibrium price ratio, p^1/p^0 . We can also see that the responsiveness of ϕ^{bP} and ϕ^{sP} to the price ratio and other variables is governed by ξ^b and ξ^s respectively. This is because ξ^b and ξ^s represent the buyer and seller's elasticity of substitution between the platforms.

We can also see what determines the fund exchange rate ratio. Exchange rates are set to make the fund indifferent about taking in new buyers. This means that the exchange rate for agents bring currency \mathcal{C} is:

$$q^{\mathcal{C}} = \frac{\partial V^f}{\partial M^{\mathcal{C}}}$$

where $M^{\mathcal{C}} := \eta^{\mathcal{C}} A$ is the fund's holdings of currency \mathcal{C} . So, the relative exchange between tokens and dollars, q^1/q^0 , is the relative willingness of the fund to accept tokens. Ultimately, the ratio q^1/q^0 is increasing in ϵ . The intuition for this result is that as the platform decreases ϵ , it decreases the usefulness of the token as a medium of exchange and so the fund offers token holders a lower exchange rate for entering the fund.

Bringing the two ratios together gives an effective "real" marginal rate of substitution between selling on the platforms:

$$\frac{q^1 p^1}{q^0 p^0}.$$

This is the exchange rate that pins down the aggregate supply of goods to the private platform through the formula for ξ^s (equation (3.5)). In this sense, to attract sellers the platform must balance increasing the price on the platform with increasing the willingness of the fund to accept the fund to accept the tokens that are used on the platform.

Using the equilibrium agent choices, we can derive the steady aggregate state variables: $\{\bar{\varphi}, \bar{\eta}, \bar{N}, \bar{A}\}$. This is done in corollary 2 below.

Corollary 2. *Any steady state $\{\bar{\varphi}, \bar{\eta}, \bar{N}, \bar{A}\}$ of the model satisfies the equations:*

$$\begin{aligned} \bar{\varphi} &= \phi^{s1}(\bar{\varphi}, \bar{\eta}), & \bar{N} &= \frac{\lambda^a}{\lambda^s}, \\ \bar{\eta} &= \phi^{b1}(\bar{\varphi}, \bar{\eta}), & \bar{A} &= \frac{M^0}{1 - \bar{\eta}}. \end{aligned} \tag{3.8}$$

Proof. See Appendix A.2.5. One might expect that the feedback effects in the model create multiple steady states but, in the appendix, we show that these forces are not always strong enough. There are regions with a unique steady state. \square

To understand this result, observe that the laws of motion for η and φ are given by:

$$\begin{aligned} d\eta_t &= \lambda^b \eta_t (\phi_t^{b1} - \eta_t) (1 - \epsilon) dt \\ d\varphi_t &= \lambda^s \frac{S_t}{B_t} (\phi_t^{s1} - \varphi_t) dt \end{aligned}$$

where S_t and B_t are the masses of sellers and buyers at time t . So, conceptually, the steady state fractions $\bar{\varphi}$ and $\bar{\eta}$ come from setting the rates of entry, ϕ^{s1} and ϕ^{b1} , equal to the rates of exit, $\bar{\varphi}$ and $\bar{\eta}$.

We can also observe that steady state aggregate wealth is a function of η while η and φ are determined without reference to steady state N or A . In this sense, the key endogenous aggregate state variables are $\{\varphi, \eta\}$. We are not able to solve for the $\{\varphi, \eta\}$ in closed form for the general case. However, there are two limiting special cases that do admit a closed form solution and are particularly helpful for understanding the forces in the model.

Special Case 1: Inelastic Buyers. The first special case is the limit as ξ^b goes to zero. Conceptually, taking the limit as $\xi^b \rightarrow 0$ makes the buyer's amenity shocks infinitely disperse and so their discrete choice problem is entirely determined by the realization of the idiosyncratic amenity shocks. In this case, the agents decisions and prices approach the limits:

$$\phi^{b1} = \bar{\eta} = \frac{1}{2}, \quad \phi^{s1} = \bar{\varphi} = \frac{1}{1 + \left(\frac{(3+\epsilon)(1+\epsilon)}{2\kappa(1+3\epsilon)}\right)^{\frac{\xi^s}{1+\xi^s}}}, \quad \frac{p^1}{p^0} = \left(\frac{2}{1+\epsilon}\right) \left(\frac{\phi^{s0}}{\phi^{s1}}\right), \quad \frac{q^1}{q^0} = \frac{1+3\epsilon}{3+\epsilon} \quad (3.9)$$

This is analogous to a Cobb-Douglas preference specification where buyers allocate a constant share of income to different consumption goods. We interpret this as the case where the buyer decisions are inelastic with respect to prices and so the general equilibrium effects in the model are minimized. If we also take the limit as $\xi^s \rightarrow 0$, then we get that seller decisions are also inelastic and so:

$$\phi^{s1} = \frac{1}{2}, \quad \frac{p^1}{p^0} = \frac{2}{1+\epsilon}. \quad (3.10)$$

Special Case 2: Elastic Buyers. At the other extreme, we can consider the case where ξ^b goes to ∞ . Conceptually, this means that the buyer no longer faces amenity shocks so their choice of platform is entirely determined by relative prices. In this case, the agents decisions and prices approach the limits:

$$\phi^{b1} \begin{cases} = 1 & \text{if } \frac{p^1}{p^0} < \frac{1+\zeta(\varphi)}{1-\eta+\eta\epsilon} \\ \in [0, 1] & \text{if } \frac{p^1}{p^0} = \frac{1+\zeta(\varphi)}{1-\eta+\eta\epsilon} \\ = 0 & \text{if } \frac{p^1}{p^0} > \frac{1+\zeta(\varphi)}{1-\eta+\eta\epsilon} \end{cases}, \quad \phi^{s1} = \frac{1}{1 + \left[\left(\frac{q^0}{\kappa q^1}\right) \left(\frac{p^0}{p^1}\right)\right]^{\xi^s}}, \quad \frac{q^1}{q^0} = \begin{cases} 1 & \text{if } \frac{p^1}{p^0} < \frac{1+\zeta(\varphi)}{1-\eta+\eta\epsilon} \\ \frac{\epsilon+1-\eta+\eta\epsilon}{2-\eta+\eta\epsilon} & \text{if } \frac{p^1}{p^0} = \frac{1+\zeta(\varphi)}{1-\eta+\eta\epsilon} \\ \epsilon & \text{if } \frac{p^1}{p^0} > \frac{1+\zeta(\varphi)}{1-\eta+\eta\epsilon} \end{cases} \quad (3.11)$$

Markets can only clear if $\phi^{b1} \in (0, 1)$ so we must have that:

$$\frac{p^1}{p^0} = \frac{1 + \zeta(\varphi)}{1 - \eta + \eta\epsilon} \quad (3.12)$$

This is analogous to a perfect substitutes preference specification where buyer decisions are infinitely elastic with respect to prices. We can interpret this as the case when general equilibrium effects are very strong.

4 Platform Competition

Industry leaders and policy makers often discuss the incentive for platforms incentives to create “walled gardens” that “lock-in” customers. This can only be fully understood if one includes a contestable market competition between the incumbent and potential entrant platform. The incumbent platform tries to “lock in” existing customers, but this is difficult with respect to the public market place, since agents anticipate that they may be locked in and hence tend to avoid the platform from the beginning. The same reasoning does not apply with respect to the competition between the incumbent and potential entrant platform. When an agent sells its output goods, they may anticipate that they are locked-in in the incumbent platform later, but agents cannot yet switch to the new entrant platform since it is not active at that point in time.

Solving the contestable markets model requires us to impose additional structure on the problem. In particular, we need to specify what would happen off-equilibrium path if new platform entry did occur. We start by describing the details of the contest between the incumbent and entrant platforms. We then set up and characterizes the solutions to the entrant and incumbent platform problem.

4.1 Additional Features of the Market Contest Model

We place a number of restrictions on the contest between the incumbent and entrant platforms to ensure that we can maintain stationarity and so keep the environment tractable. First, we “compress” the contest into one time period. We do this by imposing that if entry occurs, then all existing buyers and sellers get trading opportunities.¹⁸ This attempts to capture the idea that it takes a positive measure of trade for the entrant to capture the market. In principle, we could model this as a dynamic game where the entrant has to attract buyers for multiple period of time before dethroning the incumbent platform but we believe this introduces significant complexity without additional economic insight.

Second, we restrict attention to equilibria where the incumbent can successfully deter entry. We assume that agents have rational expectations on the equilibrium path. However, we still need to specify beliefs about the off-equilibrium path game that would be played if the entrant were to come in and contest the market. In the off-equilibrium event that the new platform enters the

¹⁸There are some subtleties in setting this up that require explanation but are not important for the economics of the problem.

market, agents believe that other agents will switch to the platform that gives sellers the higher utility.¹⁹ In addition, agents and the incumbent platform believe that, after a successful entry, the new platform brings in new buyers so that η and A immediately jump to their new steady state values.

Finally, we do not allow the entrant platform to commit to contracts with buyers and sellers about what they will do if they successfully win control of the market. Instead, we allow the entrant platform to re-optimize once they become the new incumbent platform. For convenience, we use the superscript 1 to denote the incumbent and 2 to denote the entrant.

4.2 Competitive Benchmark

We define the competitive benchmark to be the outcome in which the incumbent earns zero profit from making loans and zero profit from exchange rate fees. We characterize the competitive benchmark in lemma 1 below.

Lemma 1. *In the competitive benchmark, $\epsilon = 1$ and $\kappa^1 = \check{\kappa}$, where $\check{\kappa}$ satisfies:*

$$1 - \check{\kappa} = \left(\frac{\rho^f + \lambda^s}{q^1 p^1 \phi^{s1} \lambda^s} \right) \frac{1}{z}$$

The value of the incumbent platform is $V_0 = 0$.

Proof. See appendix A.3. □

We can rewrite this condition as:

$$(1 - \check{\kappa}) \left(\frac{\lambda^s}{\rho^f + \lambda^s} \right) q^1 p^1 \phi^{s1} = \frac{1}{z}$$

where the left-hand side is the discounted, expected repayment per loan and the right-hand side is the marginal cost making loans. We can observe that, as the cost of producing input goods goes to zero ($1/z \rightarrow 0$), the competitive share of sales revenue allocated to the incumbent platform goes zero ($1 - \check{\kappa} \rightarrow 0$).

4.3 The Entrant Problem

We solve the entrant problem by stepping “backwards” through the entry game. We first solve for the trading allocations off-equilibrium path when a new platform enters with a policy κ^2 . We then solve for whether agents would move to a new platform offering κ^2 . Finally, we set up the optimization problem for the entrant platform.

¹⁹We specify this off-equilibrium path belief to ensure a unique equilibrium. The construction of the off-equilibrium path beliefs is not without loss of generality. However, as we show later in the section, this assumption makes the problem tractable and leads to intuitive results.

Goods market and contracts after platform entry. If agents stay with the incumbent (the incumbent “wins” the fight), then the equilibrium is as described in section 3. The equilibrium price ratios satisfy equations (3.6) and (3.7), the fraction of traders on each platforms satisfies equations (3.4) and (3.5), and the allocations are summarized in table 1 below.

	Revenue ($p^P X^P$)	Seller Share	Incumbent Share	Entrant Share
Platform 0	$p^0 \lambda^s \phi^{s0} N$	1	0	0
Platform 1	$p^1 \lambda^s \phi^{s1} N$	κ^1	$1 - \kappa^1$	0
Platform 2	0	-	-	-

Table 1: Equilibrium revenue allocations if incumbent wins contest.

In the off-equilibrium path event that agents move to the new entrant platform (the entrant “wins” the fight), trading is different. The entrant persuades agents to move from the incumbent private platform to their platform so the fraction agents on the new entrant are ϕ^{b1} and ϕ^{s1} from equations (3.4) and (3.5). The market price on platform 2 is given by:

$$\frac{p^2}{p^0} = \left[\zeta (\iota \varphi)^{\xi^b (1 + \xi^b)} \left(\frac{q^0}{\kappa q^2} \right)^{\xi^s} \right]^{\frac{1}{1 + \xi^b + \xi^s}} \quad (4.1)$$

where ϵ no longer appears because the agents have to pay the exit exchange fee regardless of whether they go to the public platform or the new entrant platform and the entrant’s ability to provide the matching service depends upon the information they have about transaction histories, ι . The ratio q^2/q^0 is the relative price of entering the fund with tokens from the new platform once it has taken over the market (which will ultimately be the same as q^1/q^0 due to stationarity). The entrant must also decide what to do with the outstanding contracts. The fraction of contracts that are portable, ι , can be sold and transferred to the entrant platform. The entrant and incumbent bargain over the orphaned contracts with entrant getting a fraction $\alpha \iota (1 - \kappa^1)$ of the sales revenue on the entrant platform and the incumbent getting $(1 - \alpha) \iota (1 - \kappa^1)$. So, from the seller’s point of view, the expected sale revenue they can keep after repaying the orphaned contracts that can be enforced (but before paying any additional fees to the entrant) is: $\iota \kappa^1 + (1 - \iota)$. The entrant platform then takes $(1 - \kappa^2) (\iota \kappa^1 + (1 - \iota))$ of the seller’s revenue. The total sales revenue and allocations are summarized in table 2 below.

	Revenue ($p^{\mathcal{P}} X^{\mathcal{P}}$)	Seller Share	Incumbent Share	Entrant Share
Platform 0	$p^0 \lambda^s \phi^{s0} N$	1	0	0
Platform 1	0	-	-	-
Platform 2	$p^2 \lambda^s \phi^{s1} N$	$\kappa^2(1 - \iota + \iota \kappa^1)$	$(1 - \alpha)\iota(1 - \kappa^1)$	$\alpha \iota(1 - \kappa^1) +$ $(1 - \kappa^2)(1 - \iota + \iota \kappa^1)$

Table 2: Off-equilibrium path revenue allocations if entrant wins contest.

Agent choice to move platforms. After the new platform enters, agents who chose to search on the incumbent private platform have the opportunity to change to searching on the entrant private platform. The off-equilibrium path belief of the agents is that all agents will coordinate on the platform that maximises seller welfare. That is, buyers and sellers end up trading on the platform that solves:

$$\max_{\mathcal{P} \in \{1,2\}} \{q^{\mathcal{P}} \kappa^{\mathcal{P}} p^{\mathcal{P}}\} = \max \{q^1 \kappa^1 p^1, q^2 \kappa^2 (1 - \iota + \iota \kappa^1) p^2\}$$

Imposing the equilibrium price ratios (3.6) and (4.1), we have that the agents move to the entrant platform if and only if the sales revenue that the sellers can keep on the entrant platform is sufficiently high (see lemma 3 in appendix A.4 for working):

$$\kappa^2(1 - \iota + \iota \kappa^1) \geq \kappa^1 \frac{\zeta(\eta)}{\zeta(\iota\eta)} \frac{1}{1 - \eta + \epsilon\eta} \quad (4.2)$$

Condition (4.2) illustrates how the incumbent can make it more difficult for the entrant to attract agents. If the incumbent increases the revenue they give to sellers, $\uparrow \kappa^1$, or decreases the token to dollar conversion rate (for a given η), $\downarrow \epsilon$, then the entrant platform is forced to increase the revenue that they give, $\uparrow \kappa^2$, in order to capture the market. If the incumbent increases the portability of information, $\uparrow \iota$, then the effect on κ^2 is ambiguous because fewer sellers are able to default, $\downarrow (1 - \iota + \iota \kappa^1)$, but the entrant can improve their matching technology, $\uparrow \zeta(\iota\eta)$, and better compete for new customers. The general equilibrium impact is that increasing all of these changes also affect η . As we will see, increasing ϵ and κ typically increases η .

Entrant platform problem: We are now finally ready to set up the entrant's problem. Suppose that the incumbent platform has chosen policy $\{\kappa^1, \epsilon, \iota\}$. Suppose that the economy is currently in steady state $Z = \{\varphi, \eta, N, A\}$ satisfying (3.8) and agent choices satisfy (3.4), (3.5), (3.6), and (3.7). After observing the incumbent and agent choices, the entrant must decide whether to enter the market and convince agents to switch to their platform. Formally, the entrant solves problem

(4.3) below:

$$\max\{V^e(Z, \kappa^1), 0\} \quad (4.3)$$

where 0 is the value if the platform does not enter, and $V^e(Z, \kappa^1)$ is the platform's value if they successfully enter the market when the incumbent platform has set κ^1 . $V^e(Z, \kappa^1)$ satisfies problem (4.4) below:

$$\begin{aligned} V^e(Z, \kappa^1) &= \max_{\kappa^2} \left\{ V^e(Z^e) + q^2 \Pi^e(\kappa^2, \kappa^1) N \right\} \\ \text{s.t. } \Pi^e(\kappa^2, \kappa^1) &= \left[(1 - \kappa^2)(1 - \iota + \iota \kappa^1) + \alpha \iota (1 - \kappa^1) \right] p^2 \lambda^s \phi^{s1} \\ &+ \text{equations (4.1), (4.2)} \end{aligned} \quad (4.4)$$

where $V^e(Z)$ is the value of the entrant once they have become the new incumbent, $\Pi^e(\kappa^2, \kappa^1)N$ is the profit or loss that the entrant platform incurs from successfully contesting the market, $p^2 \lambda^s \phi^{s1} N$ is the aggregate revenue from trading on the new platform at the time that it enters the market, equation (4.1) characterizes the market equilibrium if the agents move to the new platform, condition (4.2) ensures that agents would move to the new platform, and Z^e denotes the aggregate state of the economy after new platform entry. By assumption, the platform brings new customers so that $Z^e = Z$.

Proposition 1. *The constraint on attracting sellers binds. The net income for a platform that successfully contests the market is:*

$$\Pi^e(\kappa^1) = \left[(1 - (1 - \alpha)\iota(1 - \kappa^1))(1 - \eta + \eta\epsilon) - \frac{\zeta(\eta)}{\zeta(\iota\eta)} \kappa^1 \right] \lambda^s \phi^{s1}$$

where the steady state $\eta = \bar{\eta}(\epsilon) \in [0, 1]$. The platform will not enter the market if and only if:

$$V^e(Z) \leq -q^2 \Pi^e(\kappa^1) N \quad (4.5)$$

For ξ^s sufficiently high, $\Pi^e(\kappa^1)$ is decreasing in κ^1 and so there exists a cut-off value of $\tilde{\kappa}^1$ satisfying $V(Z) = -q^2 \Pi^e(\tilde{\kappa}^1) N$ such that $\kappa^1 > \tilde{\kappa}^1$ will deter entry and $\kappa^1 < \tilde{\kappa}^1$ will not deter entry.

Proof. See appendix A.4. □

We can interpret the term $(1 - \alpha)\iota(1 - \kappa^1)(1 - \eta + \eta\epsilon)$ as the “compensation” that the entrant platform must give to the agents to offset the currency exchange cost of moving platform. Likewise, we can interpret the term $\zeta(\eta)/\zeta(\iota\eta)\kappa^1$ as the “compensation” that the entrant platform has to give to the agents to compensate them for the lower matching rate on the platform. So, the incumbent can decrease the net income of the entrant by restricting the revenue that is available when agents trade there or increasing the compensation they have to pay for a trading disadvantage.

Discussion of “lock-in” effects: The incentive of a platform to create a “walled garden” can only be understood by including contestable market competition between the incumbent and potential entrant platform. The incumbent platform tries to “lock in” existing customers, but this is difficult with respect to the public market place, since agents anticipate that they may be locked in and hence tend to avoid the platform from the beginning. The same reasoning does not apply with respect to the competition between the incumbent and potential entrant platform. When an agent sells its output goods, they may anticipate that they are locked-in in the incumbent platform later, but agents cannot yet switch to the new entrant platform since it is not active at that point in time. Equation (4.2) captures the “lock-in” and “anti-lock-in” effects forces of the incumbent platform. We have that $\zeta(\iota\eta)/\zeta(\eta) \leq 1$ because restricting transaction information makes it harder for the entrant platform to offer transaction services. We also have that $(1 - \eta + \eta\epsilon) \leq 1$ because the token “exit fees” make it costly for the buyers to take their currency to the entrant platform. So, both these effects push κ^2 above κ^1 . We interpret this as the incumbent restricting the “interoperability” of its information and tokens in order to *lock-in* in buyers and so deter entry by forcing the entrant to set a low markup in order to capture the market.

For both information portability and token exchangeability there are offsetting effects. With regard to information, we can see that $(1 - \iota + \iota\kappa^1)/\kappa^1 \geq 1$ because reducing information interoperability makes it harder for the private platform to enforce loan contracts when agents are trading on the entrant platform. This pushes κ^2 below κ^1 . We interpret this as the incumbent experiencing an *anti-lock-in effect* from making loans: an entrant can come in and give the borrowers a better default option and so allow the entrant to take more sales revenue and still capture the market. There is a different offsetting effect from restricting exchangeability. If ϵ decreases, then η can potentially decrease because sellers are less willing to hold the currency.

4.4 The Incumbent Problem

The incumbent platform chooses the steady state market equilibrium that successfully deters entry and maximises dividends to the fund. Formally, the incumbent solves problem (5.3) below:

$$\begin{aligned}
 V_0(N) = & -\frac{N}{z} + (1 - \epsilon)\eta_0q_0^0 + \max_{\kappa^1, \epsilon, \iota} \left\{ \mathbb{E}_0 \left[\int_0^\infty e^{-\rho^f t} \left(q^1(1 - \kappa^1)p_t^1 X_t^1 - \frac{\lambda^a}{z} + q^0(1 - \epsilon)(1 - \eta)p_t^1 X_t^1 \right) dt \right] \right\} \\
 \text{s.t. } & p_t^1 X_t^1 = p^1 \lambda^s \phi^{s1} N \\
 & + \text{equations (3.4), (3.5), (3.6), (3.7), (3.8), (4.5)}
 \end{aligned} \tag{4.6}$$

where N/z denotes initial equity raising to finance the initial collection of loans, $(1 - \epsilon)\eta_0q_0^0$ is the proceeds from their initial token offering, $(1 - \kappa^1)p_t^1 X_t^1 - \lambda^a$ is the net flow token income from fees, $(1 - \epsilon)(1 - \eta)p_t^1 X_t^1$ is the flow dollar seigniorage revenue from issuing tokens, and $\rho^f = \rho + \lambda^b$ is the discount rate of the fund. Conceptually, this problem says that the incumbent chooses a steady state competitive equilibrium (conditions (3.4), (3.5), (3.6), (3.7), (3.8)) to maximize fee and seigniorage

revenue subject to the constraint that new platform entry is successfully deterred (conditions (4.5)).

Discussion of the platform entry game: Although this environment is highly stylized, we believe it captures the following important features of contested markets:

- (i) It is costly for the entrant platform to successfully challenge the incumbent and take the market. In our model, this is captured by the entrant having to attract a positive measure of traders immediately after they enter and before they have been able to circulate their currency or build a customer information database. A more realistic model would model the contest as a dynamic game where the entrant gradually gains market share. However, such a model would introduce significant technical complications without illuminating additional economic mechanisms.
- (ii) The entrant platform needs to persuade buyers and sellers to move from the entrant platform after those buyers and sellers have optimized their behaviour to the incumbent platform. In our model, this is captured by the buyers and sellers choosing whether to search on the public or incumbent platform before knowing whether the entrant platform has entered the market. In this sense, we give the incumbent platform the opportunity to take advantage of buyer and seller uncertainty about new platform entry.
- (iii) The entrant platform cannot credibly promise that they will behave differently to the incumbent once they have become the monopoly private platform. In our model, this is captured by the assumption that the entrant platform cannot commit to contracts with entrants.

Discussion of currency discount rates: The platform uses q^1 and q^0 to discount token income and dollar income. This is because the incumbent platform is owned by the fund and so prices dividends using the discount rate of the fund. Ultimately, this means that the platform faces a trade-off. If the platform takes actions that devalue their token by making it a worse medium of exchange (e.g. decreasing ϵ), then they increase the discounted value of their dollar denominated seigniorage revenue but decrease the discounted value of their token fee revenue.

We look for Markov Perfect Equilibria of the contest between the incumbent and the entrant, in which the entrant chooses the same policies as the previous incumbent once they become the new incumbent. We state this in definition 2 below.

Definition 2 (Equilibrium in Platform Contest). In equilibrium, we have that $\epsilon^e = \epsilon$, $\iota^e = \iota$, $\kappa^e = \kappa$, and $V_0 = V^e$.

In following subsection, we progressively characterize the equilibrium to the platform contest. This is an involved problem since it essentially involves competition between two ‘‘Ramsey’’ platforms.

4.5 Synergies and Incumbent Market Power

If there is no regulation, the incumbent platform chooses to deviate from the competitive outcome by restricting interoperability (setting $\iota < 1$ and/or $\epsilon < 1$) so they can “lock-in” agents, deter platform entry, and earn positive profits. We state this formally in Theorem 3 below.

Theorem 3. *For sufficiently large producer productivity, z , the incumbent platform offers loans, the constraint on deterring entry binds and the incumbent platform chooses “interoperability” policies:*

$$\begin{aligned} \iota &= \arg \min_{\iota} \left\{ \Pi^e(\kappa^1, \epsilon, \iota) \right\} \\ \epsilon &< 1 \end{aligned}$$

and higher fee policy than the competitive benchmark $1 - \kappa^1 > 1 - \check{\kappa}$. The steady state value function of the incumbent platform satisfies:

$$V(Z) = -\Pi^e(\kappa^1, \epsilon, \iota) > 0$$

Proof. See appendix A.4. □

Theorem 3 says that, when the constraint on deterring entry binds, the value of the incumbent platform is the entry cost that a new platform must pay to successfully take control of the market:

$$V(Z) = \left[\frac{\zeta(\eta)}{\zeta(\iota\eta)} \kappa^1 + ((1 - \alpha)\iota(1 - \kappa^1) - 1)(1 - \eta + \eta\epsilon) \right] \lambda^s \phi^{s1} N$$

Since ι only affects the contest with the new entrant, the incumbent platform chooses ι to minimize the net income the entrant gets from successfully contesting the market, which ultimately maximizes their value. We are now well placed to return to our motivating questions.

Q. Will an incumbent platform that is bundling matching and ledger services supply uncollateralized credit? Theorem 3 says that the answer is yes. So long as z is sufficiently large, the platform will set up an uncollateralized credit market. By construction, the platform can do this because they have access to trade information about the supply chain, control of the payment technology, and the ability to incentivise use of the token ledger through the threat of exclusion. An immediate corollary is that if the platform was not providing both two services, then it could not provide uncollateralized credit. In this sense, the incumbent platform is providing a credit service that a traditional bank would not be able to provide unless they formed an information sharing partnership with a trading platform.

Q. Will an incumbent platform use their control of the digital ledger to gain market power by restricting interoperability? Again, theorem 3 says that the answer is yes. The platform will restrict currency exchange and/or information portability to earn positive profits. In this sense, the bundling of a trading platform with a digital ledger not only creates positive synergies but increases

platform rents.

The incumbent platform has many different, intersecting ways of gaining market power by restricting interoperability. To help illustrate the forces, we consider some special cases of the model that can be solved in closed form. We start by focusing on the choice of ι , then study the choice of ϵ , and finally consider a numerical solution to the overall problem.

4.5.1 Information Portability (“Ledger Competition”)

In this subsection, we focus on the trade-offs the incumbent faces when choosing the information portability between ledger. Here, the platform must balance keeping exclusive access to their transaction information (by having low information portability) and being able to enforce contracts across ledgers (by having high information portability)

To understand the platform’s key trade-offs involved with choosing information portability, we consider a special case of the environment where there the incumbent is only strategically competing with the entrant platform and their only tool is restricting information portability. Formally, we impose that there is no exchange rate fee, $\epsilon = 1$, and the idiosyncratic amenity shocks are infinitely disperse, $\xi^b = \xi^s = 0$. Not having an exchange rate fee means that money and tokens are equally useful as media of exchange so there is no “currency competition”. Having infinitely disperse amenity shocks means that the buyers and sellers are completely inelastic and so the equilibrium satisfies the characterization from equations (3.9) and (3.10). Conceptually, we can think about this as shutting down the competition from public marketplace so that there is only competition between with the entrant platform.

For this simplified environment, the platform problem reduces to choosing κ^1 and ι to pick the equilibrium that maximizes loan revenue subject to the constraint on deterring entry.

Corollary 3 (Unregulated Market). *For sufficiently large z , $\alpha > 0$, and $\zeta(1/2) > 0$, the constraint on deterring entry binds, the platform chooses:*

$$\iota = 0, \quad 1 - \kappa^1 = \frac{1 - \check{\kappa} + \frac{\rho^f}{\lambda^b \lambda^s} \zeta(1/2)}{2 + \frac{\rho^f}{\lambda^b \lambda^s} (1 + \zeta(1/2))} > 1 - \check{\kappa},$$

and the incumbent platform has strictly positive initial value:

$$V_0 = \frac{\frac{\zeta(1/2)}{1 + \zeta(1/2)} - \left(\frac{\rho^f + \lambda^s}{\lambda^b \lambda^s} \right) \frac{1}{z}}{\frac{\rho^f}{\lambda^b \lambda^s} + \frac{2}{1 + \zeta(1/2)}} > 0$$

Proof. See appendix A.4 □

The incumbent platform chooses ι to minimize the cost of entry, which for this special cases

becomes:

$$\iota = \arg \min_{\iota} \left\{ -(1 - \alpha)(1 - \kappa^1)\iota - (1 + \zeta(\iota/2))\kappa^1 \right\}.$$

Substituting in $\iota = 0$ and $\iota = 1$, we can see that the incumbent chooses $\iota = 0$ if:

$$(1 + \zeta(1/2))\kappa^1(0) - \kappa^1(1) > (1 - \alpha)(1 - \kappa^1(1))$$

where $\kappa^1(\iota)$ denotes the minimum κ^1 that deters entry when ι is chosen. The left hand side of this expression is the additional compensation that the entrant must pay to compensation agents for a lower matching technology when $\iota = 0$ while the right hand side is the additional share of contract enforcement that the incumbent can receive when $\iota = 1$. It turns out that $\kappa^1(1) > 1$ because in this case the incumbent has no transaction history advantage but most attempt to deter an entrant who can “steal” a fraction of their loan contracts. If $\alpha = 0$ and $\zeta(1/2) = 0$, then the incumbent platform is indifferent between $\iota \in \{0, 1\}$, although both generate negative profits. As these terms become positive, the incumbent wants to set $\iota = 0$ so the platform can use their transaction history advantage to offset the loss that comes from the entrant stealing their loan book.

4.5.2 Token Exchangeability (“Currency” Competition)

In this subsection, we focus on the trade-offs the incumbent platform faces when choosing the exchange rate between tokens and money. The platform must balance making the token an attractive medium of exchange (by setting a high exchange rate) with being able to extract seigniorage revenue (by setting a low exchange rate). This is similar to the “conventional” currency design problem in traditional money models. The novel feature of the environment is that the platform also wants to use tokens to lock customers into their network (by setting a low exchange rate) and deter new entry.

To highlight these trade-offs in the formal model, we consider a special case of the environment in which there is full information portability, $\iota = 1$, the incumbent platform takes all profit on orphaned contracts, $\alpha = 1$, and there is no externality from transaction histories, $\zeta(\varphi) = 1$. This environment shuts down a key component of “ledger competition”, in the sense that information portability is not being chosen strategically. We consider the two extreme cases of inelastic buyer demand and elastic buyer demand.

Inelastic buyer demand: We start by considering the case in which the buyer’s idiosyncratic amenity shock is infinitely disperse, $\xi^b = 0$. This makes buyer demand inelastic to prices because the buyer’s amenity shocks are infinitely disperse and so their discrete choice problem is entirely determined by the realization of the idiosyncratic amenity shocks. For this case, the equilibrium is characterized by (3.9).

In order to successfully contest the market, the entrant must choose:

$$1 - \kappa^2 = - \left(\frac{1 - \epsilon}{1 + \epsilon} \right) \leq 0$$

which gives entrant net income during the contest of:

$$q^2 \Pi^e N = - \left(\frac{1 - \epsilon}{1 + \epsilon} \right) \kappa^1 (1 + 3\epsilon^2) \frac{\lambda^b}{4} N \leq 0.$$

Both inequalities hold with equality at $\epsilon = 1$. So, by forcing the entrant to compensate the buyers for paying currency exit costs, the incumbent forces the entrant to make a loss on attracting agents.

For this simplified environment, the incumbent platform problem reduces to choosing κ^1 and ϵ to pick the equilibrium that maximizes loan revenue subject to the constraint on deterring entry:

$$V_0 = - \frac{N}{z} + \frac{(1 - \epsilon)(3 + \epsilon)}{4(1 + \epsilon)} + \max_{\kappa^1, \epsilon} \left\{ \int_0^\infty e^{(\rho + \lambda^b)t} \left[\left((1 - \kappa^1) \left(\frac{1 + 3\epsilon}{1 + \epsilon} \right) + \frac{(1 - \epsilon)(3 + \epsilon)}{4(1 + \epsilon)} \right) \frac{\lambda^b}{2} - \frac{1}{z} \right] \lambda^s N dt \right\}$$

$$s.t. \quad V^e \leq \left(\frac{1 - \epsilon}{1 + \epsilon} \right) \kappa^1 (1 + 3\epsilon^2) \frac{\lambda^b}{4} N$$

We can see the different effects of decreasing ϵ . First, it decreases the value of the token loan fee revenue:

$$\downarrow q^1 (1 - \kappa^1) p^1 X^1 = \downarrow (1 - \kappa^1) \left(\frac{1 + 3\epsilon}{1 + \epsilon} \right) \frac{\lambda^b \lambda^s N}{2}$$

This is because tokens can now less easily be exchanged into dollars and so have become less useful as a medium of exchange. Consequently, the fund entry rate for sellers coming with tokens, q^1 , decreases and so the value of platform token revenue falls. Second, decreasing ϵ increases the seigniorage revenue that the incumbent can earn

$$\uparrow q^0 (1 - \epsilon) (1 - \eta) p^1 X^1 = \uparrow \frac{(1 - \epsilon)(3 + \epsilon)}{4(1 + \epsilon)} \frac{\lambda^b \lambda^s N}{2}$$

This is partly because the platform is now promising a low conversion rate and so does not have to hold as many dollars to back the currency. In the limit as $\epsilon \rightarrow 0$, the platform is essentially issuing a fiat currency. It is also partly because the fund's valuation of the dollars that are received from seigniorage goes up, $\uparrow q^0$. Finally, decreasing ϵ also increases the cost of entry:

$$\uparrow \left(\frac{1 - \epsilon}{1 + \epsilon} \right) \kappa^1 (1 + 3\epsilon^2) \frac{\lambda^b}{4} N$$

and so relaxes the constraint on deterring entry. The incumbent must balance these effects when choosing ϵ . For this special case, our problem admits a closed form solution, which is characterized in corollary 4 below.

Corollary 4 (Token Exchangeability With Inelastic Demand). *For z sufficiently large, the constraint on deterring entry binds and the platform chooses:*

$$\epsilon = 0, \quad 1 - \kappa^1 > 1 - \check{\kappa}$$

and the incumbent platform has strictly positive initial value:

$$V_0 > 0$$

Proof. See appendix A.4 □

Why does the incumbent platform set $\epsilon = 0$ but not $\kappa^1 = 0$? Decreasing ϵ and decreasing κ^1 both lead to higher fee revenue. However, they have opposite effects on the deterring entry. Decreasing ϵ lowers entrant profit and so deters entry while lowering κ^1 increases entrant profit and so encourages entry. This means that increasing κ^1 is always “cheaper” than increasing ϵ . This effect is reinforced by the change in q since q^1 decreases as ϵ decreases, making in the real value of low token fee revenue lower. So, corollary 4 illustrates the power of the “lock-in” effect when general equilibrium effects are minimized: the incumbent platform is able to deter entry and increase fee revenue at the same time.

Elastic buyer demand: We now consider the other extreme, in which $\xi^b = \infty$. In this case, buyer demand is infinitely elastic with respect to prices because the buyer does not receive amenity shocks. The equilibrium is characterized by (3.11) and (3.12). This means that ϕ^{b1} and η are now given by:

$$\eta = \phi^{b1} = \phi^{s1} = \frac{1}{1 + \left(\frac{q^0 p^0}{\kappa q^1 q^0}\right)^{\xi^s}}, \quad \frac{p^1 q^1}{p^0 q^0} = \frac{\frac{\epsilon}{1-\eta+\eta\epsilon} + 1}{2 - \eta + \eta\epsilon}$$

So, now decreasing ϵ as an additional general equilibrium impact on the fraction of currency held in tokens, η , and the volume of trade on the platform, $\lambda^s \phi^{s1} N$. Ultimately, this implies that the incumbent platform sets $\epsilon > 0$.

Corollary 5 (Token Exchangeability with Perfectly Elastic Demand). *For z sufficiently large, the constraint on deterring entry binds and the platform chooses $\epsilon > 0$.*

Proof. See Appendix A.4. □

As soon as η and ϕ^{b1} depend upon ϵ , the solution is difficult to characterize. For this reason, we study a numerical solution to the overall problem.

4.5.3 Numerical Illustration

The overall model cannot be solved in closed form. However, we can illustrate the solution numerically. This is done in figure 1, which plots the equilibrium for different values of x_i^b . As we know

from subsection , at $\xi^b = 0$ the platform sets $\epsilon = 0$ because they are able to decrease ϵ without affecting aggregate demand. In fact, at $\xi^b = 0$, the platform actually sets $\kappa > 0$ because they are very willing to take a loss on their loan contracts in order to earn seigniorage revenue from currency sales. However, as ξ^b increases, the elasticity of η with respect to ϵ increases and eventually the incumbent start to increase ϵ and decrease κ below 1. We can also see that eventually the value of the incumbent (the top right) goes negative. This is because for high ξ^b , a productivity of 10.0 is insufficient to guarantee a positive overall platform value.

5 Policy Evaluation

In this section, we use our digital ledger model to understand and evaluate a collection current proposals. We start by studying “open banking” regulation. We then consider interoperability regulation more broadly. Finally, we consider different possible implementations of CBDC, with and without a programmable public ledger. Ultimately, we show that regulation is only welfare improving if it is able to preserve the underlying synergies that come from combining matching services, digital ledgers, and credit services.

5.1 Information Portability Regulation and “Open Banking”

There has been much recent discussion about introduction of “open banking” legislation that allows agents, rather than financial intermediaries, to control the portability of their financial information. Regulators, particularly in countries that have implemented open banking, have expressed a strong prior that such regulation will be welfare improving. Bankers have opposed the regulation arguing that it will prevent them from offering credit. In this section, we use our model to show to understand the two positions. Ultimately, we show that open banking not necessarily an optimal arrangement.

To focus on the first order impact of open banking, we return to the special case from section 4.5.1. However, we now consider an environment where it possible to separately choose the portability of transaction histories that are used for improving the matching technology, ι^h , and the portability of contract terms that are used for enforcing contracts, ι^c . In this sense, the information portability choice has been “disaggregated”.

We start with the problem of the incumbent platform. As before, the incumbent platform chooses information portability to maximize the cost of new platform entry. So, with the disaggregated information portability choice, the platform solves:

$$\iota = \arg \min_{\iota^h, \iota^c} \left\{ \hat{\Pi}^e(\kappa^1, \iota^h, \iota^c) = \frac{1}{2} \left(1 - (1 - \alpha)(1 - \kappa^1)\iota^c - \frac{1 + \zeta(1/2)}{1 + \zeta(\iota^h/2)} \kappa^1 \right) \right\}$$

We can interpret the term $\kappa^1 \zeta(1/2) / \zeta(\iota^h/2)$ as reflecting the “discount” that the entrant platform gives to the agents for having an inferior matching technology. This term is minimized for $\iota^h = 0$ since this gives entrant the maximum information disadvantage. We can interpret $(1 - \alpha)(1 - \kappa^1)\iota^c$

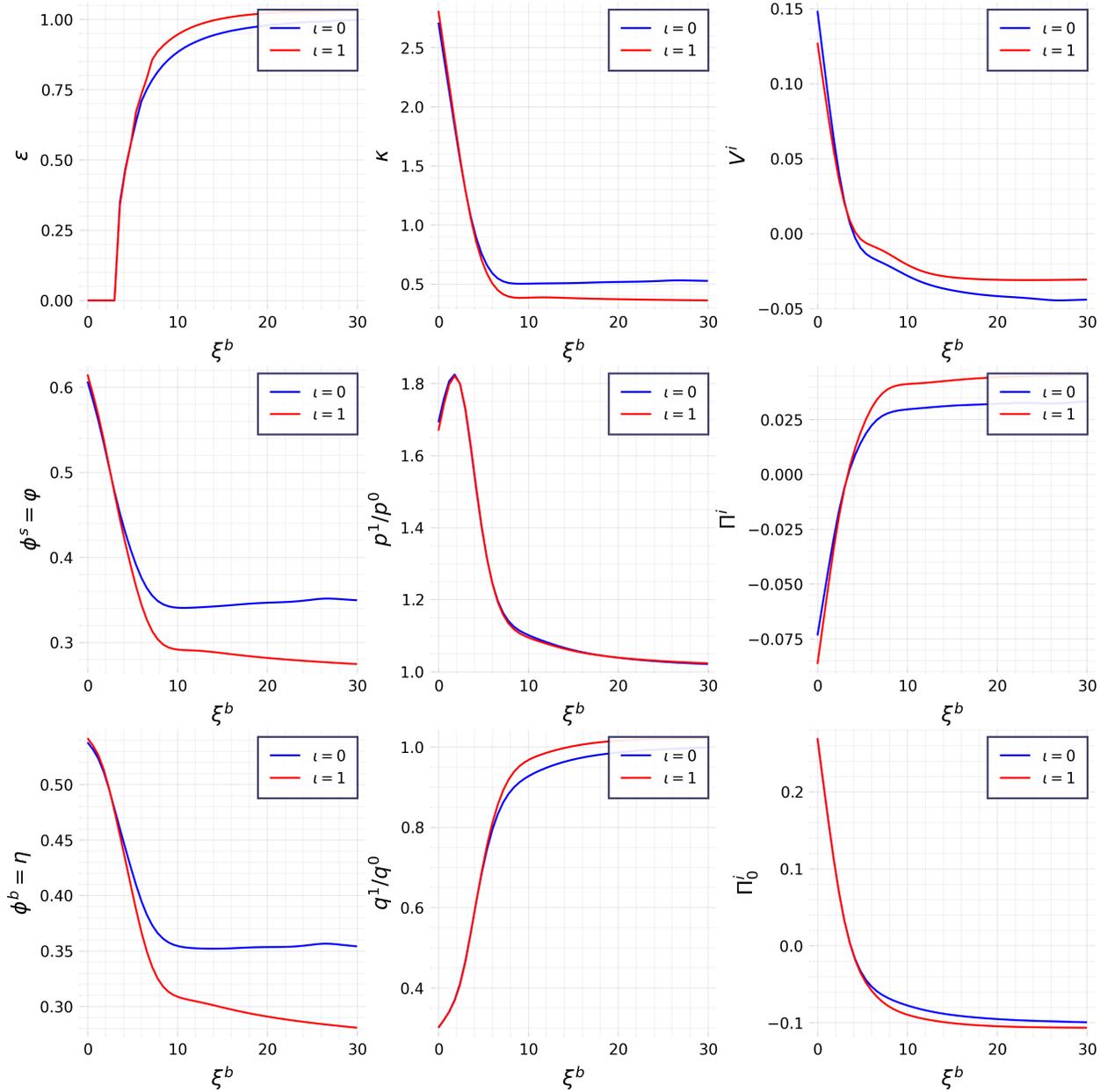


Figure 1: Equilibrium for Various Values of ξ^b .

The figures show numerical solutions to the platform game for $\xi^b \in [0, 30]$. The other parameter values are $z = 10.0$, $\rho = 0.1$, $\xi^s = 1.0$, $\lambda^a = \lambda^s = \lambda^b = 0.5$, $\zeta(\varphi) = 0.05\varphi^{0.05}$, $\alpha = 0.5$. The platform problem is solved to a tolerance of 10^{-14} . There are 51 grid points for ξ^b . A spline with smoothing parameter 4.0 has been applied to the results.

as the “discount” that the entrant gives the sellers for having to repay their existing contracts when then move the to the new platform. This term is minimized at $\ell^c = 1$ since then all contracts can be enforced once the sellers move to the new platform. So, the entry cost is maximized for the combination $\ell^h = 0$ and $\ell^c = 1$. At the other extreme, the entry cost is minimized for the entry cost is minimized for the combination $\ell^h = 1$ and $\ell^c = 0$, in which case the entrant can take all the

income that the incumbent would have been able to get:

$$\hat{\Pi}^e(\kappa^1, 1, 0) = \frac{1}{2}(1 - \kappa^1)$$

These observations lead to corollaries 6 and 7 below.

Corollary 6 (Hidden Ledger). *Suppose the incumbent platform can separately choose ι^h and ι^c . Suppose that z is sufficiently large that the constraint on deterring entry binds. Then, the incumbent chooses:*

$$\iota^h = 0; \quad \iota^c = 1, \quad 1 - \kappa^{h1} = \frac{1 - \check{\kappa} + \frac{\rho^f}{\lambda^b \lambda^s} \zeta(1/2)}{2 + \frac{\rho^f}{\lambda^b \lambda^s} (\alpha + \zeta(1/2))} > 1 - \kappa^1 > 1 - \check{\kappa}^1$$

which leads to value:

$$V^s = \frac{\frac{\zeta(1/2)}{\alpha + \zeta(1/2)} - \left(\frac{\rho^f + \lambda^s}{\lambda^b \lambda^s} \right) \frac{1}{z}}{\frac{\rho^f}{\lambda^b \lambda^s} + \frac{2}{\alpha + \zeta(1/2)}} > V > 0$$

where κ^{h1} and V^h refers to the contract terms κ^1 and value function for a hidden ledger.

Proof. See appendix A.5. □

We interpret this arrangement as similar to the current information portability system under the current banking system. Banks maintain hidden ledgers and have considerable discretion over which information is shared. They block information about transactions to maintain their comparative advantage in service provision but share information about contract enforcement and default through a credit registry system. This system maximises the incumbent platform's ability to deter entry and moves the economy further away from the competitive market than under the transparent ledger model.

An alternative to both the current system and the centralized, transparent ledger system we model this paper, is to give agents control over the sharing of their information. This is often referred to as an "open banking" system and has implemented in some countries. In the context of our model, we interpret this as regulation that gives agents control over the portability of their information. For our purposes, we assume that buyers and sellers decide as separate, collective groups about whether to port information. Buyers choose whether to port transaction histories, ι^h , since they generated the information on the ledger. Sellers choose whether to port contract information, ι^c , since they wrote the contracts.

Corollary 7 (Open Banking). *If the buyers and sellers choose information portability, then they choose:*

$$\iota^h = 1; \quad \iota^c = 0$$

and the profit maximising incumbent platform chooses:

$$1 - \kappa^{o1} = \frac{1 - \check{\kappa}}{2 + \frac{\rho^f}{\lambda^b \lambda^s}} < 1 - \check{\kappa}$$

which leads to value $V_0^o < 0$ where κ^{o1} and V_0^o refers to the contract terms and value function under open banking. This implies that no incumbent platform would set up an uncollateralized credit market.

Proof. See Appendix A.5. □

This corollary emphasises that buyers and sellers have the opposite exposure to information on the platform. Buyers have a “long” position in the information because it allows new entrant platforms to provide a better matching service if they switch to the entrant. As a result, they choose to port the information. Sellers have a “short” position with respect to the information on the ledger because it enables their contracts to be enforced. As a result, they choose to not port the information. The overall outcome is the worst possible combination for the incumbent platform because it maximises the net income that an entrant platform can earn. Ultimately, no platform would want to set up the uncollateralized market. In this sense, open banking breaks the synergy between retail matching, ledger provision, and lending. This means that the economy reduces back to a regular banking model with collateralized credit.

Intuitively, we can think about the provision of uncollateralized credit as a public good in the market. Uncollateralized credit is needed to start the supply chain. However, once credit is set up, entrant platforms can come in and “steal” customers by offering the possibility to default on the new ledger. Giving agents complete control over the portability of information does not allow the incumbent to earn the profit required to compensate them for setting up the credit market. We summarize these results in table 3.

	Perfect Comp	Contested Markets	Hidden Ledger	Open Banking
ι		$\iota = 0$	$\iota^h = 0, \iota^c = 1$	$\iota^h = 1, \iota^c = 0$
$1 - \kappa^1$	$\left(\frac{\rho^f + \lambda^s}{\lambda^b \lambda^s}\right) \frac{2}{z}$	\uparrow	$\uparrow\uparrow$	\downarrow
V	0	> 0	$\gg 0$	< 0

Table 3: Summary of market outcomes under different regulatory regimes.

From table 3, it is clear that information portability extremes do not deliver the competitive allocation. In principle, the regulator could choose an intermediate level of information portability, $\iota \in (0, 1)$, and implement the competitive equilibrium. However, from a practical point of view, it is not clear that it makes sense to consider an intermediate ι . In the next section, we add exchange fees back into the problem, which gives an alternative way of allowing the incumbent platform to recoup the costs of setting up the credit supply chain.

5.2 Interoperability Regulation

As emphasized by this paper, information portability is only one feature of interoperability. Regulators have also expressed a desire to enforce complete interoperability between platforms. An important implication of our model is that a credit market is not sustainable under full interoperability because there is no way to reward the incumbent platform for setting up the market. This is stated in corollary 8.

Corollary 8. *Full interoperability ($\iota = 1, \epsilon = 1$) shuts down the credit market. For $\iota = 1$, there exists $\epsilon \in (0, 1)$ such the competitive equilibrium is restored.*

Proof. Follows immediately from Theorem 3. □

In order to restore the competitive equilibrium, the incumbent needs to give the platform a way to earn back the strategic disadvantage that comes from setting up the market. If they face a restriction that $\iota \in \{0, 1\}$, then they can do this by setting $\iota = 1$ and then allowing the incumbent platform to charge exit fees and earn seigniorage revenue.

5.3 Central Bank Digital Currency

Instead of regulating private tokens and interoperability, the government can also compete with private platform tokens by offering a Central Bank Digital Currency (CBDC). One primary objective of CBDC is to avoid tokenization since it limits government's monetary sovereignty, i.e. to maintain the power to conduct monetary policy and secure seigniorage revenue.

A major potential downside of introducing CBDC is that it might disrupt synergies in the banking system. Other papers have discussed liquidity management synergies across deposits and credit lines (Kashyap et al. (2002), Piazzesi and Schneider (2020)) and the potential for bankers to have superior investment opportunities (Keister and Sanches (2019)). We focus on new synergies that come from the platform having control of the payment ledger, access to trade information, and the ability to exclude agents from trade. As we have shown, bundling these services allows the platform to supply uncollateralized credit that a traditional bank would not be able to provide. In this subsection, we show that different implementations of CBDC may enhance or destroy this synergy depending on whether the CBDC competes as just a currency provider or also competes as a programmable ledger.

5.3.1 Digital Legal Tender CBDC

Thus far, we have assumed that tokens are accepted on the private platform and dollars are accepted in the public marketplace. However, an important potential feature of CBDC is that it might act as a digital legal tender. To explore this policy, we now consider an environment in which sellers can always choose to accept digital dollars (CBDC) on any platform. That is, sellers can now choose to accept payment in dollars when trading with buyers on the private platform.

In this new environment, if the platform wants the sellers to accept a particular currency, then it must offer a contract that incentivizes this choice. Formally, the platform now offers a currency contingent contract specifying that the platform gets a fraction $1 - \kappa^{\mathcal{P}\mathcal{C}}$ of sales revenue if seller trades on platform \mathcal{P} and accepts currency \mathcal{C} from the buyers. We let $1 - \bar{\kappa}^{\mathcal{P}\mathcal{C}}$ denote the maximum enforceable value of $1 - \kappa^{\mathcal{P}\mathcal{C}}$. We restrict attention to contracts that can be enforced. This imposes the constraint that:

$$1 - \kappa^{\mathcal{P}\mathcal{C}} \leq 1 - \bar{\kappa}^{\mathcal{P}\mathcal{C}}. \quad (5.1)$$

In the absence of regulation on acceptability, $1 - \bar{\kappa}^{\mathcal{P}\mathcal{C}}$ is given by:

$$1 - \bar{\kappa}^{\mathcal{P}\mathcal{C}} = \begin{cases} 0, & \text{if } (\mathcal{P}, \mathcal{C}) \in \{(0, 0), (1, 0), (2, 0)\} \\ 1, & \text{if } (\mathcal{P}, \mathcal{C}) \in \{(1, 1), (2, 1)\} \\ \iota, & \text{if } (\mathcal{P}, \mathcal{C}) = (2, 2) \end{cases}$$

These constraints are formalizing the following enforcement constraints in the model. If the seller trades using dollars on either platform, $(\mathcal{P}, \mathcal{C}) \in \{(0, 0), (1, 0), (2, 0)\}$, then the incumbent platform does not control the payment flow and so cannot enforce any repayment. If the seller trades using tokens, then the incumbent platform controls the payment flow and so can enforce repayment. Finally, if the seller trades using entrant tokens, then incumbent's enforcement power depends upon the interoperability of the contracts.

Corollary 9 (CBDC as a pure Legal Tender). *If the government introduces CBDC on a separate, disconnected ledger, then there is no equilibrium in which the incumbent platform provides uncollateralized credit.*

Proof. This follows immediately the setup. Since $1 - \bar{\kappa}^{10} = 0$, the sellers will always choose $\mathcal{C} = 0$ and default. \square

Conceptually, a legal tender CBDC eliminates the incumbent platform's ability to provide uncollateralized credit because buyers and sellers can undertake "side-trades" using dollars on the private platform's matching service. The platform can see that agents have matched but does not control the flow of payments and so cannot enforce the contracts, even when the sellers use their platform.

5.3.2 "Smart" CBDC

An alternative implementation is for the government to offer a legal tender CBDC bundled with a transparent, programmable ledger controlled by the Fed that competes directly with any privately provided digital ledgers. We refer to such a ledger as "smart" CBDC ledger. In this case, even if CBDC becomes the dominant currency, the incumbent platform could use the CBDC ledger to write and enforce contracts. However, this requires the incumbent platform to provide information to the CBDC ledger. In this sense, the choice of information "portability", ι , becomes a choice of

information “provision” to the CBDC ledger. As before, providing information to the digital ledger allows the incumbent platform to better enforce contracts if the entrant comes into the market but also gives the entrant platform transaction information that they can use to provide matching services. The difference is that now the contract information provided by the incumbent platform can also be used to enforce contracts on the public marketplace because trades are taking place on the CBDC ledger.

Formally, this implies that the enforceability constraint, $1 - \bar{\kappa}^{\mathcal{P}\mathcal{C}}$, becomes:

$$1 - \bar{\kappa}^{\mathcal{P}\mathcal{C}} = \begin{cases} 1, & \text{if } (\mathcal{P}, \mathcal{C}) \in \{(0, 1), (1, 1), (2, 1)\} \\ \iota, & \text{if } (\mathcal{P}, \mathcal{C}) = \{(0, 0), (1, 0), (2, 0), (2, 2)\} \end{cases} \quad (5.2)$$

That is, the incumbent platform can choose enforceability by choosing ι . This implies that the incumbent platform now solves:

$$\begin{aligned} V_0(N) = & -\frac{N}{z} + (1 - \epsilon)\eta_0 q_0^0 + \max_{\kappa^1, \epsilon, \iota} \left\{ \mathbb{E}_0 \left[\int_0^\infty e^{-\rho^f t} \left(q^1(1 - \kappa^{11})p_t^1 X_t^1 + q^0(1 - \kappa^{10})p_t^0 X_t^0 - \frac{\lambda^a}{z} \right. \right. \right. \\ & \left. \left. \left. + q^0(1 - \epsilon)(1 - \eta)p_t^1 X_t^1 \right) dt \right] \right\} \\ \text{s.t. } & p_t^1 X_t^1 = p^1 \lambda^s \phi^{s1} N \\ & + \text{equations (3.4), (3.5), (3.6), (3.7), (3.8), (4.5), (5.2), (5.2)} \end{aligned} \quad (5.3)$$

The new term, $q^0(1 - \kappa^{10})p_t^0 X_t^0$, is the revenue that the incumbent platform gets from enforcing contracts on the public marketplace. If the upper bound on $1 - \kappa^{10}$ binds, then the new term is given by:

$$q^0(1 - \kappa^{10})p_t^0 X_t^0 = q^0 \iota p_t^0 X_t^0$$

so the incumbent platform’s choice of information portability determines enforcement on the public marketplace.

Corollary 10. *For sufficiently small $\zeta(1)$, there is an equilibrium in which the incumbent platform chooses $\iota = 1$ and successfully enforces all contracts through the CDDBC ledger.*

Proof. See Appendix A.7. □

Conceptually, corollary 10 says tells us that there is potentially a positive externality from the provision of “smart” CDDBC ledger. The government has an advantage in getting their digital currency used across platforms. As a consequence, they can offer a programmable ledger where contracts can be enforced across a wide range of digital markets. In this sense, the introduction of “smart CBDC” turns the problem into one of incentivizing platforms to provide information onto a common ledger.

6 Discussion

In this final section, we discuss how this paper connects to topical issues relating to digital currencies that are not explicitly included in our model.

Unbundling the digital token ledger from the platform trade ledger. We model a platform that provides both a matching service and token ledger. Alternatively, we could consider a model in which the matching service and the token ledger are provided by separate entities. In this case, it would only be possible to provide uncollateralized credit if the matching platform and the token ledger were able to contract to share information. We leave this contracting problem for future research.

Intermediation. Our model assumes that contracts can only be enforced through a transparent, programmable ledger. This means that, as soon as side trades are possible, the incumbent platform stops providing credit. However, in reality, platforms could also choose directly intermediate payments. If we expanded the model in this way, then the introduction of a legal tender CBDC would change the market structure of the economy by encouraging many more platforms to intermediate more transactions. Of course, setting up a payment intermediation system potentially brings other costs and trade-offs for the platform. We leave the task of modeling intermediation to future work.

Decentralization versus transparency. In recent years, a large industry has emerged that uses decentralized blockchains with “smart” contract technology to provide financial services without intermediaries (often referred to as “DeFi”). In our paper, we abstract from the costs and benefits of decentralized blockchains and instead focus exclusively on centrally controlled ledgers. We make this choice because there are already many papers that discuss the issues around decentralization and because the scalability challenges of decentralized systems currently make them very difficult to deploy across the entire economy. However, if the scalability problems are resolved, then decentralization could potentially be an alternative approach to interoperability regulation for limiting platform market power.

Tokenization. We have considered the possibility that CBDC is used on the private platform (“dollarization”). The other extreme would be if the tokens ended up being used on the public marketplace (“tokenization”). In our model, the platform can only get sellers to accept tokens on the public marketplace by giving them the same share of revenue they would get from default. From the platform’s perspective, this exactly offsets the benefit of broader token acceptance. So, purely from the point of view of contract enforcement, there are no economies of scale and the incumbent platform is indifferent about tokenization. However, this changes once we consider the interaction with other platform services. If the incumbent can get more agents to use their tokens, then they can more effectively “lock” customers into their network. Thus, the incumbent platform would be willing to give generous loan contracts that incentivize token acceptance.

Privacy and anonymity. In our model, the platform was able to link the identities of sellers to the identity of producers. This is because no seller has inventory without a debt contract to the platform so the platform can simply do this by forcing sellers to post not only an offer to sell a good but also the debt contract that they will repay. In a more complex model where some agents have inventory without a debt contract, the platform would need to incentivize agents to reveal their identity. More generally, our model suggests that there are additional complex tradeoffs when it comes to privacy provision. The major synergy in this paper, the provision of uncollateralized trade credit, can only occur if sellers are denonymized (although buyers could still be anonymous). This suggests there is a strong trade-off between exploiting the full potential of digital ledgers and protecting privacy.

Multiple platforms and Smart CBDC. As we discussed, the introduction a Smart CBDC turns the problem of credit enforcement into one of incentivizing information sharing with the CBDC ledger. In our model, we assumed that the public marketplace would provide information with the CBDC ledger and then considered whether the incumbent platform would also want to provide information. In reality, we would need to consider how many different incumbent platforms might coordinate on information sharing with the digital ledger. We leave this problem for future work.

7 Conclusion

In this paper, we model the strategic decision making of a platform that provides three services: matching in the goods market, token money creation, and credit extension. We use this model to explore what happens when “BigTech” platforms start providing “FinTech” services. We find that Tech platforms can expand uncollateralized credit across a supply chain by exploiting synergies between the different services that they provide. However, Tech platforms will also use their control of the ledger to increase their market power by restricting the portability of some information and the exchangeability of tokens across ledgers. How should policy makers respond? Interoperability regulation and offering a CBDC alternative to provide private tokens should limit private rent extraction. However, this will only restore the competitive equilibrium if they are able to maintain the synergy benefits that the platform provides. Ultimately, implementing CBDC can only get the right balance if it provides both a competing token and a competing programmable ledger with the capacity to store information. This suggests that we need further work on the viability of a “smart”, programmable, CBDC ledger.

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A Non-Credit Economy: Supplementary Proofs

A.1 Discrete Choice Problems

This section of the appendix contains working for the discrete choice problems. Since these are standard results, we provided limited detail.

Lemma 2. *Let $\{\zeta^n\}_{n \leq N}$ be a collection of independent draws from $Gu(\xi, \mu)$, where $\mu = -\xi\mathcal{E}$ and \mathcal{E} represents the Euler–Mascheroni constant. Let $u(c) = \log(c)$. Then:*

$$\max_{n \leq N} \{\zeta^n + \varphi^n u(\pi^n)\} \sim Gu\left(\xi, \mu + \xi \log\left(\sum_n (\pi^n)^{\varphi^n/\xi}\right)\right) \quad (\text{A.1})$$

and so we have:

$$\begin{aligned} \mathbb{E}[\max_n \{\zeta^n + \varphi^n \log(\pi^n)\}] &= \xi \log\left(\sum_n (\pi^n)^{\varphi^n/\xi}\right), \\ \mathbb{P}\left(n = \operatorname{argmax}_{n'} \{\zeta^{n'} + \varphi^{n'} \log(\pi^{n'})\}\right) &= \frac{(\pi^n)^{\varphi^n/\xi}}{\sum_{n'} (\pi^{n'})^{\varphi^{n'}/\xi}} \end{aligned}$$

Proof. Using the definition of the Gumbel distribution and the independence of the N draws, we have that:

$$\begin{aligned} \mathbb{P}(\max_n \{\zeta^n + \varphi^n u(\pi^n)\} \leq k) &= \prod_n \mathbb{P}(\zeta^n + \varphi^n u(\pi^n) \leq k) \\ &= \exp\left(\sum_n -e^{-(k-\mu)/\xi} e^{\varphi^n u(\pi^n)/\xi}\right) \\ &= \exp\left(-e^{-(k-\mu-\xi \log(\sum_n e^{\varphi^n u(\pi^n)/\xi}))/\xi}\right) \end{aligned}$$

which implies result (A.1). From the properties of the Gumbel distribution, the expectation is:

$$\begin{aligned} \mathbb{P}\left(n = \operatorname{argmax}_{n'} \{\zeta^{n'} + \varphi^{n'} \log(\pi^{n'})\}\right) &= \left[\mu + \xi \log\left(\sum_n (\pi^n)^{\varphi^n/\xi}\right)\right] + \xi\mathcal{E} \\ &= \xi \log\left(\sum_n (\pi^n)^{\varphi^n/\xi}\right) \end{aligned}$$

and the probability of choosing n is:

$$\begin{aligned} \mathbb{P}(n = \operatorname{argmax}\{\zeta^{ni} + \varphi^n \log(\pi^n)\}) &= \frac{e^{\varphi^n u(\pi^n)/\xi}}{\sum_{n'} e^{\varphi^{n'} u(\pi^{n'})/\xi}} \\ &= \frac{(\pi^n)^{\varphi^n/\xi}}{\sum_{n'} (\pi^{n'})^{\varphi^{n'}/\xi}} \end{aligned}$$

□

A.2 Equilibrium

In this section of the appendix, we characterize the market equilibrium defined in section 3.2. We start by solving the buyer and seller problems. We then use the solution to the buyer problem to solve the problem of the fund. We then solve for the equilibrium prices. Finally, we derive the laws of motion for the state variables.

A.2.1 Solution to Individual Agent Problems

The aggregate state variables are $Z_t = \{\varphi_t, \eta_t, N_t, A_t\}$, where φ_t is the fraction of current buyers who sold on the private platform, η_t is the fraction of fund currency in tokens, N_t is total inventory in the economy, and A_t is total currency in the fund. Let $V^b(a, Z)$ denote the value function of a buyer with individual wealth a when the aggregate state of the world is Z . Let $V^s(n, Z)$ denote the value function of a seller with inventory n when the aggregate state of the world is Z . Since we have restricted attention to stationary equilibria with constant Z , we drop the time subscripts throughout this section.

Proof of Theorem 1. We start with the solution to the buyer's problem. The buyers' cash in advance constraint binds so buyer consumption is given by:

$$c^{\mathcal{P}} = \frac{(1 - \eta + \epsilon^{1\mathcal{P}}\eta)a}{p^{\mathcal{P}}}, \quad \forall \mathcal{P} \in \{0, 1\}$$

Thus, the HJBE for the buyer is given by:

$$\begin{aligned} \rho V^b(a, Z) &= \partial_a V^b(a, Z) \hat{\mu}^a(Z) a + D_Z V^b(a, Z) \mu^Z(Z) \\ &\quad + \mathbb{E} \left[\max_{\mathcal{P}} \left\{ \lambda^b \left(\zeta^{b\mathcal{P}i} + \log(c^{\mathcal{P}}) - V^b(a, Z) \right) \right\} \right] \\ &= \partial_a V^b(a, Z) \mu^a(Z) a + D_Z V^b(a, Z) \mu^Z(Z) \\ &\quad + \lambda^b \left(\mathbb{E} \left[\max_{\mathcal{P}} \left\{ \zeta^{b\mathcal{P}i} + \log \left(\frac{1 - \eta + \epsilon^{1\mathcal{P}}\eta}{p^{\mathcal{P}}} \right) \right\} \right] + \log(a) - V^b(a, Z) \right) \end{aligned}$$

Lemma 2 implies that the fraction of buyers who choose \mathcal{P} is given by:

$$\phi^{b\mathcal{P}} = \frac{\left(\frac{\zeta^{\mathcal{P}}(1-\eta+\epsilon^{1\mathcal{P}}\eta)}{p^{\mathcal{P}}} \right)^{\xi^b}}{\sum_{\mathcal{P}'} \left(\frac{\zeta^{\mathcal{P}'}(1-\eta+\epsilon^{1\mathcal{P}'}\eta)}{p^{\mathcal{P}'}} \right)^{\xi^b}}$$

and the HJBE becomes:

$$\begin{aligned} \rho V^b(a, Z) &= \partial_a V^b(a, Z) \mu^a(Z) a + D_Z V^b(a, Z) \mu^Z(Z) \\ &\quad + \lambda^b \left(\log \left(\bar{v}^b(A) \right) + \log(a) - V^b(a, Z) \right) \end{aligned}$$

where:

$$\bar{v}^b(A) := \left(\sum_{\mathcal{P}} \left(\frac{\zeta^{\mathcal{P}} (1 - \eta + \epsilon^{1\mathcal{P}} \eta)}{p^{\mathcal{P}}} \right)^{\xi^b} \right)^{1/\xi^b}$$

We guess and verify that, in the stationary equilibrium, the value function takes the form:

$$V^b(a, Z) = v^{ba}(Z) \log(a) + v^{b0}(Z)$$

Substituting this functional form into the HJBE and equating coefficients gives that:

$$\begin{aligned} v^{ba} &= \frac{\lambda^b}{\rho + \lambda^b} \\ v^{b0} &= \left(\frac{\lambda^b}{\rho + \lambda^b} \right) \left(\frac{\mu^a}{\rho + \lambda^b} + \log(\bar{v}^b(A)) \right) \end{aligned}$$

which gives the desired result.

We now consider the problem of the seller. Their HJBE is given by:

$$\begin{aligned} \rho V^s(n, Z) &= \partial_Z V^s(n, Z) + \mathbb{E} \left[\max_{\mathcal{P}} \left\{ \lambda^s \left(\zeta^{s\mathcal{P}j} + V^b(q^{\mathcal{P}} \kappa^{\mathcal{P}} p^{\mathcal{P}} n, Z) - V^s(n, Z) \right) \right\} \right] \\ &= \partial_Z V^s(n, Z) + \lambda^s \left(\zeta^{s\mathcal{P}j} + \mathbb{E} \left[\max_{\mathcal{P}} \left\{ \zeta^{s\mathcal{P}j} + v^{ba}(Z) \log(q^{\mathcal{P}} \kappa^{\mathcal{P}} p^{\mathcal{P}} n) \right\} \right] + v^{b0}(Z) - V^s(n, Z) \right) \end{aligned}$$

Lemma 2 implies that the fraction of sellers who choose \mathcal{P} is given by:

$$\frac{\left(q^{\mathcal{P}} \kappa^{\mathcal{P}} p^{\mathcal{P}} n \right)^{\xi^s}}{\sum_{\mathcal{P}'} \left(q^{\mathcal{P}'} \kappa^{\mathcal{P}'} p^{\mathcal{P}'} n \right)^{\xi^s}}$$

and their HJBE becomes:

$$\rho V^s(n, Z) = D_Z V^s(n, Z) + \lambda^s \left(v^{ba}(Z) \log(\bar{v}^s(Z)) + v^{ba}(Z) \log(n) + v^{b0}(Z) - V^s(n, Z) \right)$$

where $\bar{v}^s(Z)$ is given by:

$$\bar{v}^s(Z) = \left(\sum_{\mathcal{P}} \left(q^{\mathcal{P}} \kappa^{\mathcal{P}} p^{\mathcal{P}} n \right)^{\xi^s} \right)^{1/\xi^s}$$

We guess and verify that, in the stationary equilibrium, the value function takes the form:

$$V^s(n, Z) = v^{sn}(Z) \log(n) + v^{s0}(Z)$$

Substituting this functional form into the HJBE and equating coefficients gives that:

$$\begin{aligned}v^{sn} &= \left(\frac{\lambda^s}{\rho + \lambda^s}\right) \left(\frac{\lambda^b}{\rho + \lambda^b}\right) \\v^{s0} &= \left(\frac{\lambda^s}{\rho + \lambda^s}\right) \left(\log\left((\bar{v}^s(Z))^{\xi^s} (\bar{v}^b(Z))^{\xi^b}\right) + \frac{\mu^a}{\rho + \lambda^b}\right)\end{aligned}$$

which gives the desired result. □